Wave propagation in force chains

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ABSTRACT: We present numerical simulations of acoustic wave propagation in disordered granular media. Small amplitude oscillations of compressed static configurations of frictionless spherical grains are analyzed. Force chains, while important for the static properties of the granular assemblies, do not play a dominant role in transmitting acoustic waves according to our observations. Reasons include that the stiffness network is less widely distributed than the force network, and the nearby grains are coupled relatively strongly to grains in the force chain. The transmitted acoustic signal can be decomposed to an initial cycle and a more noisy tail, similarly to related experiments.

1 INTRODUCTION

It is known for quite long time now that the inter-grain forces exhibit strong spatial fluctuations in quasi-static granular piles, even though the packing density is relatively uniform. In 2 dimensions both experimental and numerical evidence shows exponential probability distribution of the number of contacts supporting large forces near the rigidity threshold, and Gaussian tails were found at higher pressures (Makse et al. 2000). Results in 3 dimensions are similar to those in 2 dimensions.

The inter-grain contacts that support large forces tend to be arranged in long linear structures. This is because a contact transmitting large force on one side of a grain is usually balanced with a single large force contact on the other side, and this repeats at neighboring grains at contact. These *force chains* are important: their rigidity or fragileness determine the macroscopic mechanical properties of the granular assemblies. The chains tend to form networks, with linear segments of characteristic length. The linear segments are supported by weak forces from the sides to prevent buckling.

Experimentally the force chains have been seen in 2 dimensional packings using photoelastic disks (Howell et al. 1999). Observing them in 3 dimension however is more difficult task. On the other hand, if probing them with acoustical excitation would reveal information about their characteristics, then the method of acoustic probing would provide an easy experimental access to them.

In this paper we present preliminary results investigating the role of force chains, in the absence of friction, in the conduction of acoustic waves. If we consider a one-dimensional array of grains, there the group velocity of waves is higher if the array is compressed more (Coste et al. 1997). Similarly, a simple calculation shows that in homogeneous elastic media (with a hard block embedded in a soft medium) the small amplitude acoustic waves propagate principally in the hard material. From these observations one expects that the acoustical waves first traveling through the granular medium are transmitted by the strong force chains. In this paper we address the question, whether the higher sound velocity of strong force chains survives in disordered media in higher dimensions.

2 THE MODEL

To investigate these questions, we performed numerical experiments on 2 and 3 dimensional granular packings, although this paper contains results for 2 dimensional packings only. We prepare a static configuration of grains, then send an acoustic pulse through the sample, and study its propagation.

We start the preparation by filling up a container with spherical grains with polydispersity $\pm 10\%$ in radius. The boundary conditions are periodic in the horizontal direction and rigid walls at the top and bottom. Then it is compressed under (vertical) uniaxial stress.

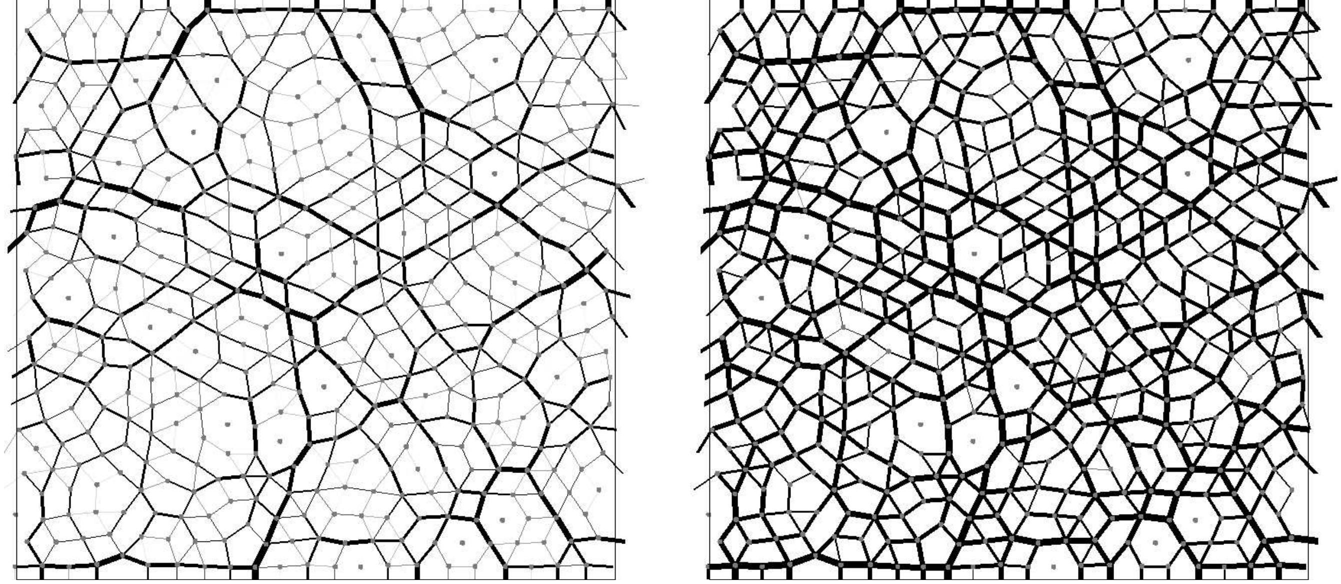

Figure 1: The static configuration for a typical 2 dimensional configuration. (a) Force chains. The width of the lines connecting two grain centers are proportional to the absolute value of the force between the grains. (b) Stiffness network, line width proportional to stiffness of link.

The grains interact via Hertzian forces. For the results presented in this paper we did not include friction, and later in the dynamics we have no dissipation. To keep the pressure uniform in the sample, gravitational force is not applied on the grains. The mechanical equilibrium configuration is obtained by energy minimization techniques. The mean coordination number of the grains is around 4, with most grains having 3, 4 or 5 contacts. The equilibrium ratio of the vertical and horizontal principal stress in the sample is close to unity.

After the equilibrium configuration has been found, we linearize small amplitude oscillations around it: the inter-grain contacts are replaced by linear springs, with stiffness obtained from the differential stiffness of the Hertzian contacts at the equilibrium configuration.

We calculate the motion by solving the eigenproblem of the linear spring system. After obtaining the amplitude of the eigenmodes by projecting the initial condition onto them, the subsequent motion is calculated by the superposition of the eigenmodes multiplied by these amplitudes.

For initial conditions, the grains in contact with the bottom wall of the container get an initial velocity at $t = 0$. This is equivalent with raising the bottom wall by a small amount for a short time, and lowering it back.

3 NUMERICAL RESULTS

A typical 2-dimensional configuration containing 500 grains is shown on Figure 1. The pressure in this sample was low, close to the rigidity threshold.

The amplitude of the acoustic wave at an early time is depicted on Figure 2. As can be seen from the figure, the grains exhibiting large amplitude oscillations do not follow closely the strongest force chains.

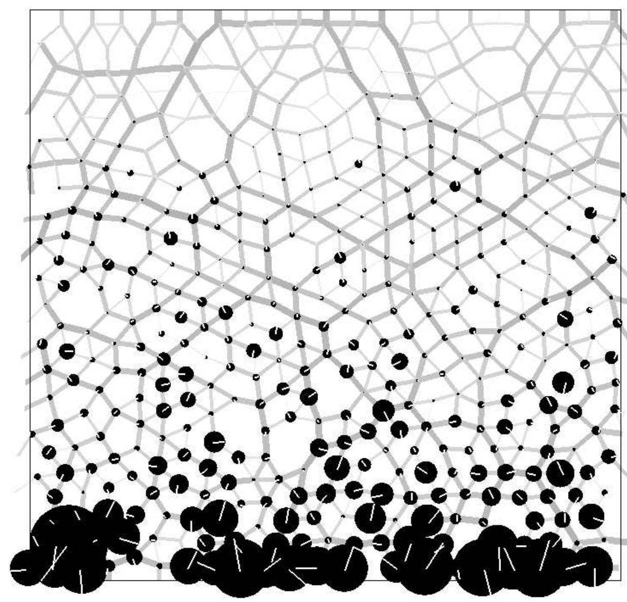

Figure 2: Acoustic wave: oscillation amplitude at time $t = 100$. Circle diameters are proportional to vibration amplitude, with white sticks indicating the direction.

We also measured the force exerted by the moving grains (in addition to the static force) on the top wall, see Figure 3a. The Fourier spectrum of this force is shown on Figure 3b. The large peaks in the spectrum correspond to strongly excited eigenmodes of the system. The eigenmode corresponding to one of the peaks ($\omega = 0.288$) is on Figure 4.

It is interesting to note, that the first cycle of oscillations appears to be of lower frequency than the rest of the signal (the frequency corresponding to this cycle is shown with arrow on Figure 3b). The separa-

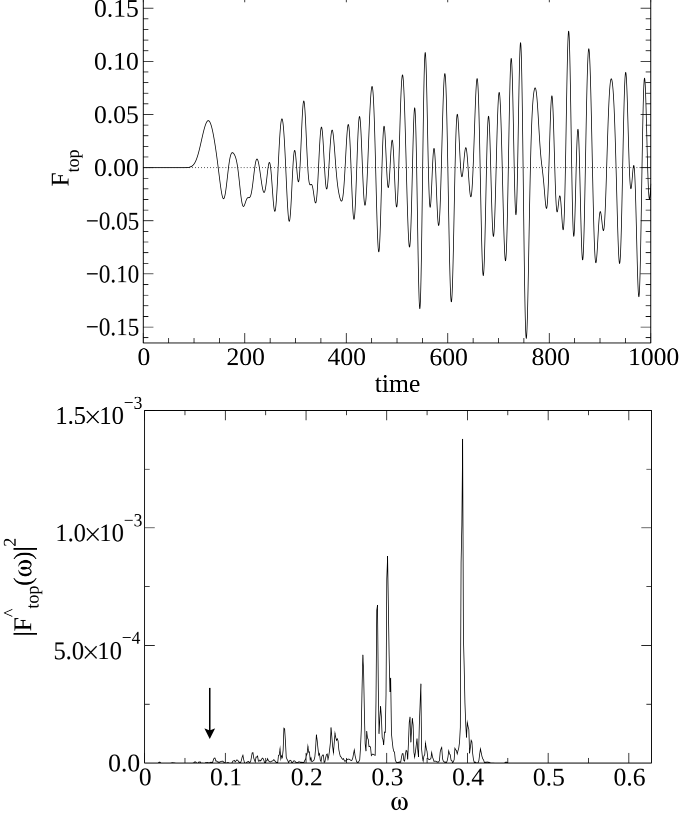

Figure 3: (a) Extra force caused by the moving grains on the top wall. Note that the apparent frequency of the first cycle of the signal is different from the typical frequency of the following tail. (b) Fourier transform of the force. The arrow denotes the frequency of the first cycle of the signal. The peaks correspond to strongly excited eigenmodes of the system.

tion of the force signal to an early coherent part and a later noisy part has been observed in an experiment on glass beads (Jia et al. 1999).

4 DISCUSSION

One of the reasons why it is not the strongest force chains that conduct the waves is that the stiffness network, determining the oscillations, is more homogeneous than the force network. For Hertzian forces, the stiffness is proportional to the cubic root of the force, making the ratio of the strongest to average link much closer to unity than the same quantity for forces.

It can also be seen that the coupling of the motion of grains in the force chain with neighboring grains is strong in 2 dimensions. Therefore nearby grains quickly pick up the amplitude of oscillation, smearing out the chains. Interconnects of the force chains contribute to this effect.

Finally, we mention that in this calculation the static frictional force between grains was not included. The inclusion of this effect would provide an additional coupling between the grains, potentially changing the behavior of the oscillations.

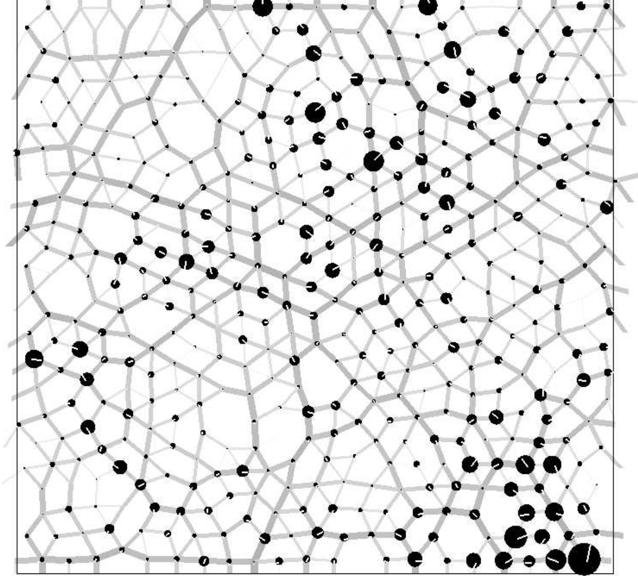

Figure 4: One of the strongly excited eigenmodes, $\omega = 0.288$. Notation is same as in Figure 2.

We have not addressed the interesting question of large amplitude oscillations. In a one-dimensional array the nonlinear excitations lead to soliton waves (Coste et al. 1997), which could have counterparts in higher dimensions. In addition, larger amplitude oscillations could lead to a rearrangement of the underlying static configuration.

5 CONCLUSIONS

We presented a numerical study of acoustic wave propagation in disordered granular media. We investigated linearized small amplitude oscillations of the grains around the equilibrium position in compressed static configurations. Contrary to expectations, observations show that force chains do not play a dominant role in transmitting acoustic waves. The time dependent transmission signal of a mechanical pulse can be decomposed to an initial low frequency cycle and a more noisy tail, as seen in experiments.

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