Beyond the Standard Model

Laboratory physics applied to the whole Universe: summary.

✓ Laws of gravitation over the whole Universe ⇒ expansion of the Universe. Hubble law

✓ Laws of thermodynamics ⇒ Hot Big Bang theory.

✓ Atomic physics. Thomson scattering ⇒ properties of cosmic microwave background radiation

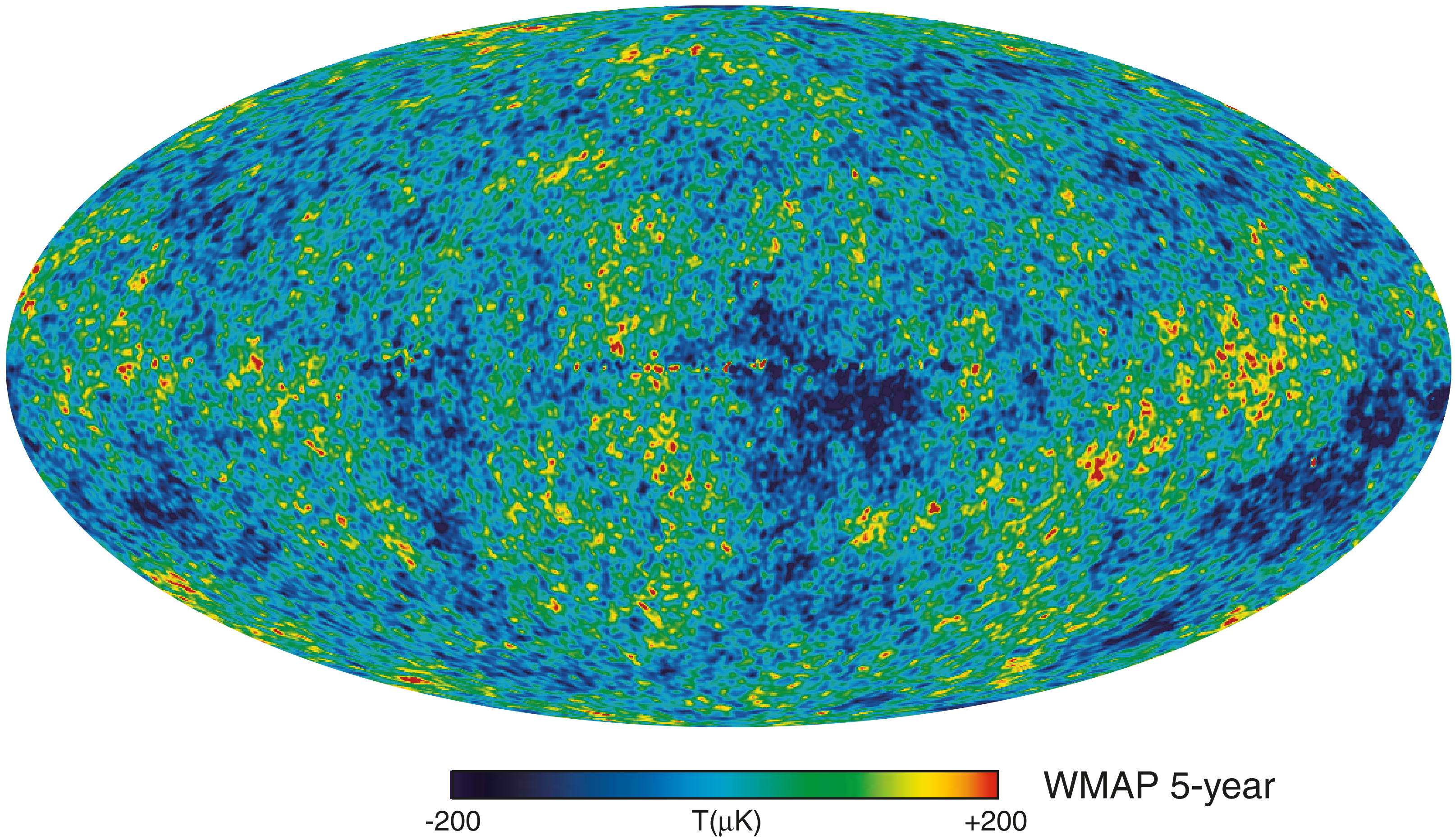
✓ Nuclear physics. Nuclear cross-section. Binding energy ⇒ Primordial synthesis of elements. Sensitive to details of the SM. ”Cosmic chronometer”.

✓ Particle physics. Weak interactions (Fermi theory) ⇒ decoupling of neutrinos. Neutral currents are important! ($e^+ + e^- \to \nu + \bar{\nu}$)

✓ Particle physics of the curved space time ⇒ inflationary theory. Generation of primordial perturbations

Precision cosmolgy!

CMB temperature is anisotropic over the sky with $\delta T/T_{\text{CMB}} \sim 10^{-5}$



WMAP-5 results with subtracted galactic contribution (courtesy of WMAP Science team)

- The temperature anisotropy $\delta T(\hat{n})$ is expanded in spherical harmonics $Y_{lm}(\hat{n})$:

$$\delta T(\vec{n}) = \sum_{l,m} a_{lm} Y_{lm}(\hat{n})$$

- a_{lm}'s are Gaussian random variables (before sky cut)
- CMB anisotropy **(TT) power-spectrum**: 2-point correlation function

$$\boxed{\langle \delta T(\hat{n})\, \delta T(\hat{n}') \rangle = \sum_{l=0}^{\infty} \frac{2l+1}{4\pi} C_l P_l(\hat{n}\cdot\hat{n}')}$$

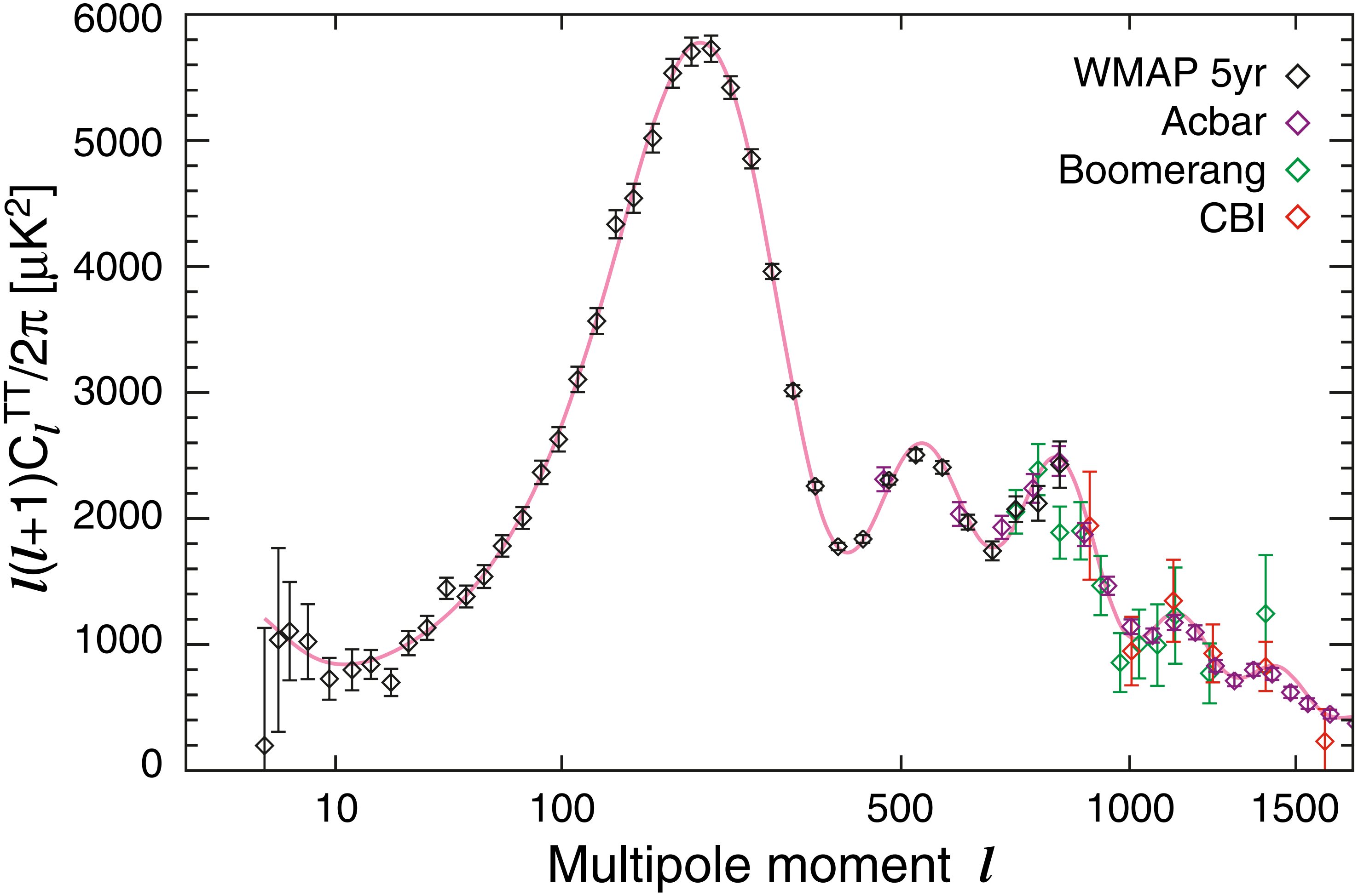
  $P_l(\hat{n}\cdot\hat{n}')$ – Legendre polynomials

- **Multipoles C_l's**

$$\boxed{C_l = \frac{1}{2l+1} \sum_{m=-l}^{l} |a_{lm}|^2}$$

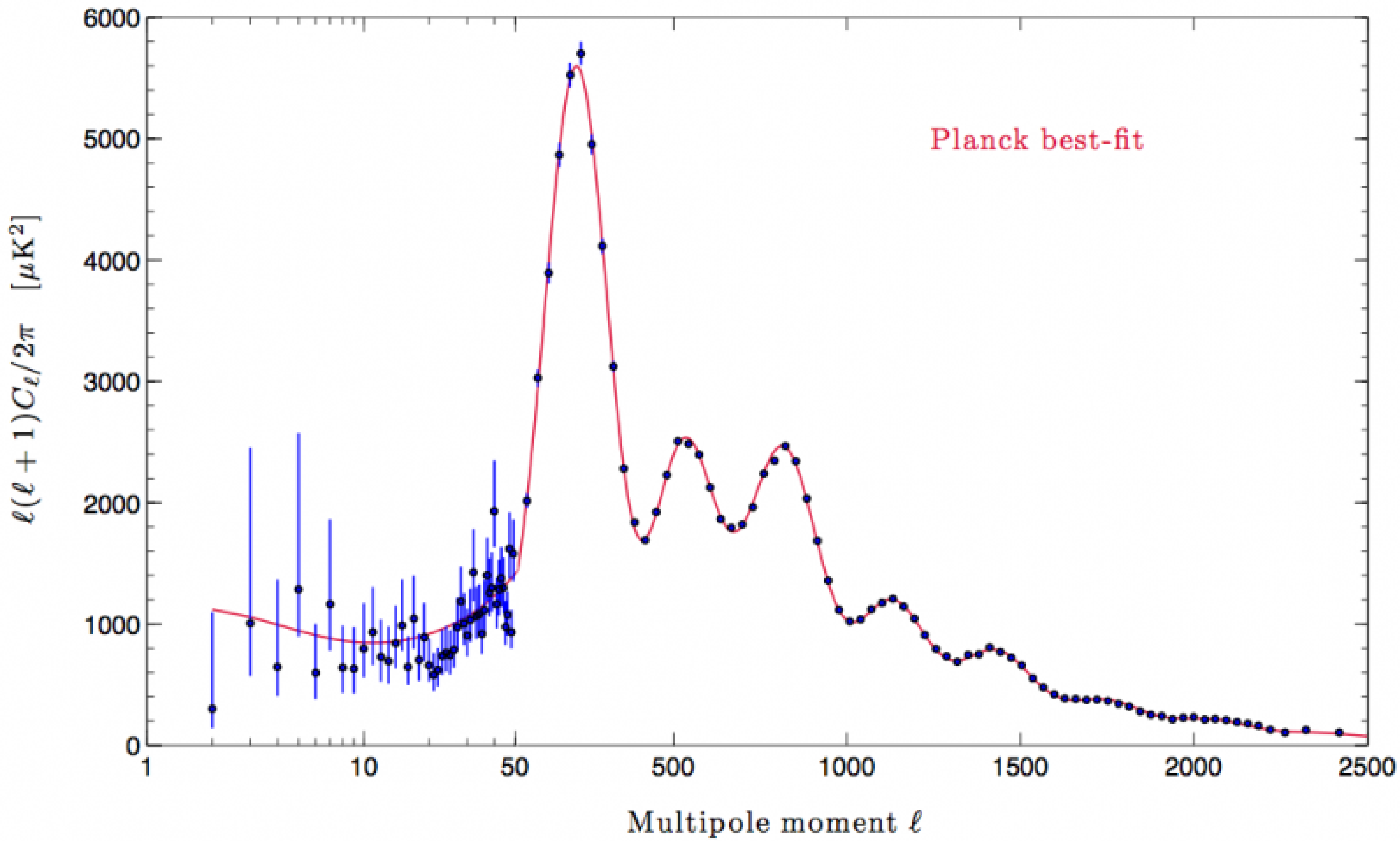
  probe correlations of angular scale $\theta \sim \pi/l$

WMAP + small scale experiments



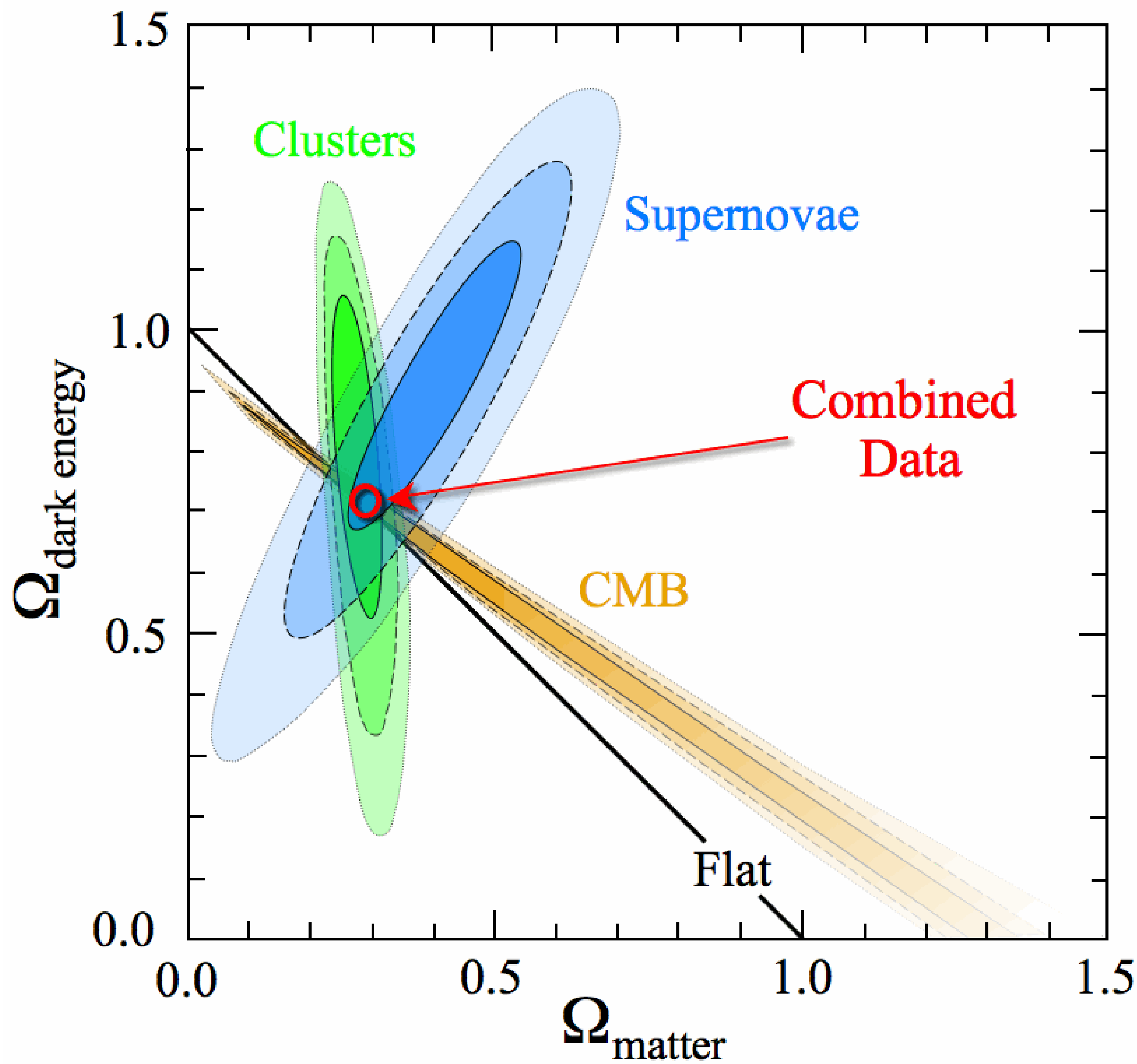
The WMAP 5-year TT power spectrum along with recent results from the ACBAR (Reichardt et al. 2008, purple), Boomerang (Jones et al. 2006, green), and CBI (Readhead et al. 2004, red) experiments. The red curve is the best-fit ΛCDM model to the WMAP data.

Is this a success?

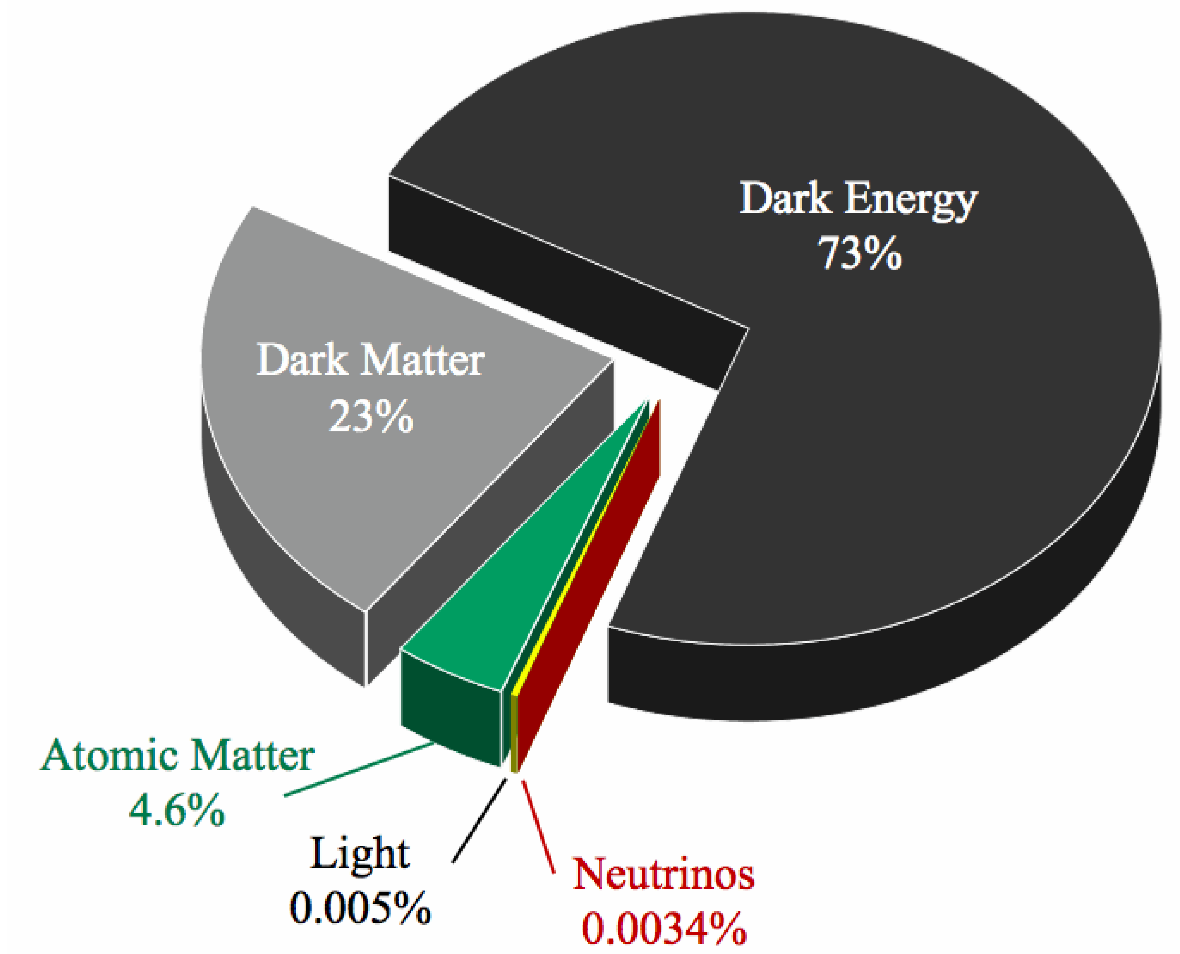


- Starting from $\ell \sim 100$ or so we do not even see error bars on the data points
- Yet the model successfully predicts all the wiggles

Is this a success?



Is this a complete success?



- We understand only about 5% of the total composition of the Universe

- Why is our universe devoid of **anti-matter**? What violated symmetry between particles and anti-particles in the early Universe?
- What is **Dark Matter** that accounts for some 86% of the total matter density in the Universe and have driven the formation of structure in the early Universe?
- What drives **inflation**?

BSM problems

- We learned that neutrinos **oscillate** from the solar physics. Neutrinos also contribute to the matter balance of the Universe. Their disappearance and and then re-appearance in a different form and their masses require new particles.

It is easy (theoretically :-)) to create its own solution for each problem

- Dark matter particle:
  - assume heavy neutral particle, not interacting with the Standard Model.
  - Assume its coupling to something in the very early Universe, e.g. produced when inflaton decays

  Not a physical model – makes no predictions

- Baryon asymmetry of the Universe:
  - Assume a particle X that decays $X \to \bar{q} + \bar{\ell}$ and $X \to q + q'$
  - Assume that X freezes out non-relativistic (a la WIMP) and then decays
  - Assume CP-violation in the processes $X \to qq$ and $X \to \bar{q}\bar{q}$

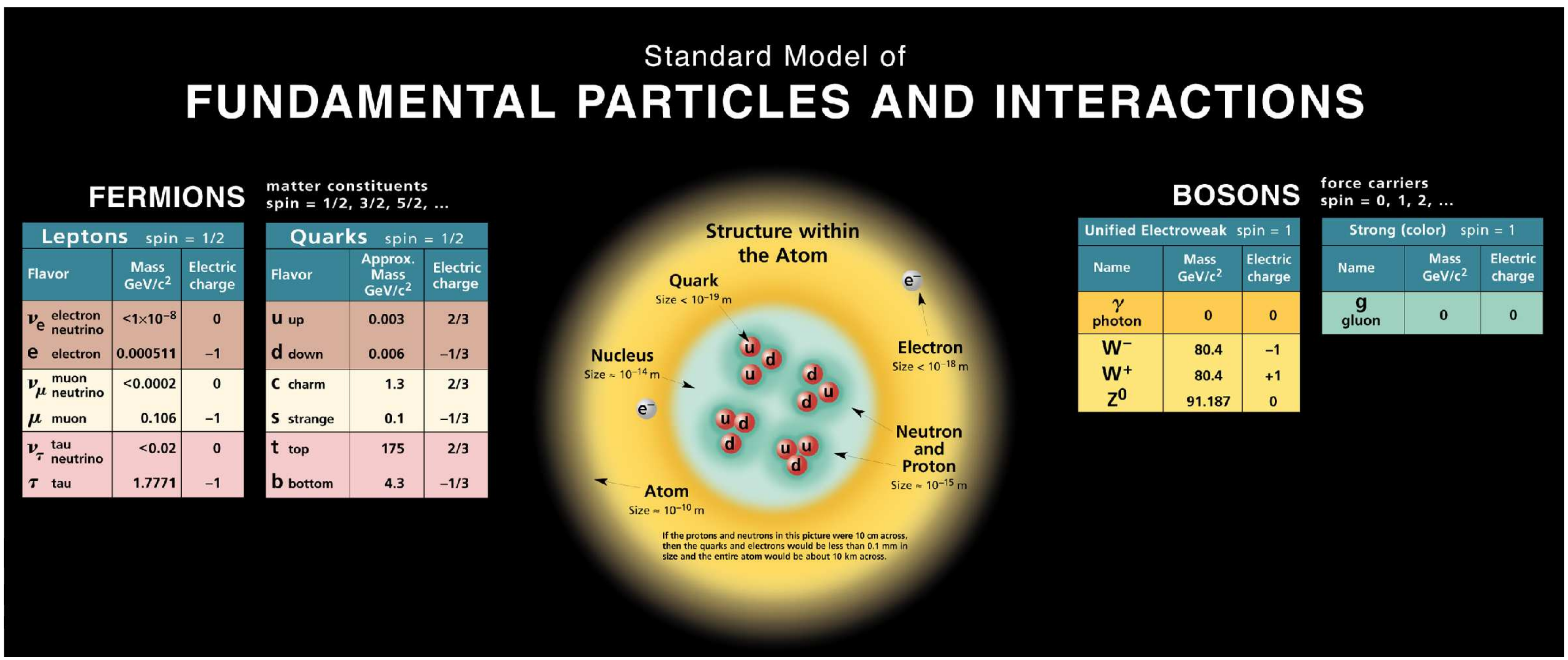
- Good baryogenesis scenario (all numbers may be made to work), but again **not testable**

A model that would allow to solve not one but several problems with few assumptions ("Okkam's razor").

Testable predictions

Are there any problems in particles physics?

Standard Model of particle physics



Leptons spin = 1/2		
Flavor	Mass GeV/c^2	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0
e electron	0.000511	−1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	−1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	−1

Quarks spin = 1/2		
Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.003	2/3
d down	0.006	−1/3
c charm	1.3	2/3
s strange	0.1	−1/3
t top	175	2/3
b bottom	4.3	−1/3

Unified Electroweak spin = 1		
Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.4	−1
W^+	80.4	+1
Z^0	91.187	0

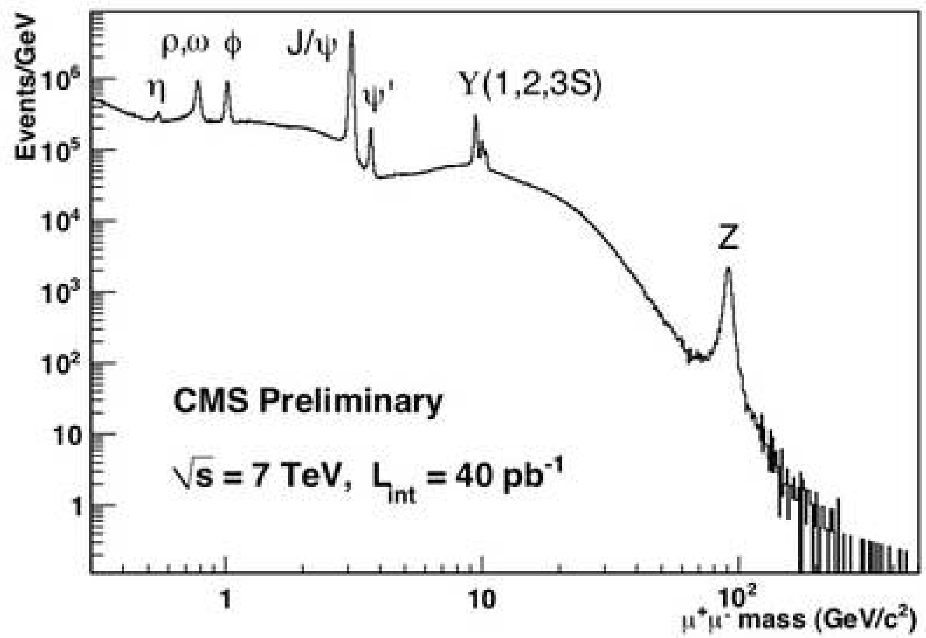
Strong (color) spin = 1		
Name	Mass GeV/c^2	Electric charge
g gluon	0	0

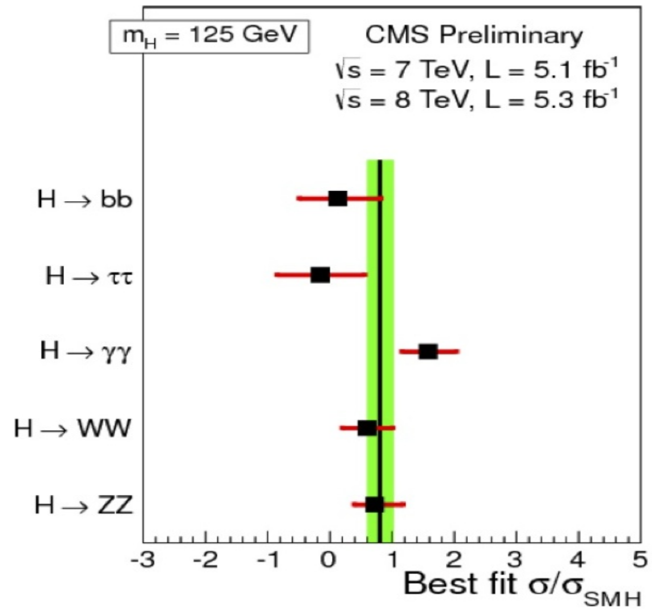
Standard Model: defined by gauge symmetry & multiplet content

gauge group	$SU(3)$ $\times$	$SU(2)$ $\times$	$U(1)_Y$ $\times$	gravity
bosons	G_μ^A	W_μ^I	B_μ	$g_{\mu\nu}$
	H^α			
fermions	$q_L,\quad u_R,\quad d_R,\quad \ell_L,\quad e_R$			

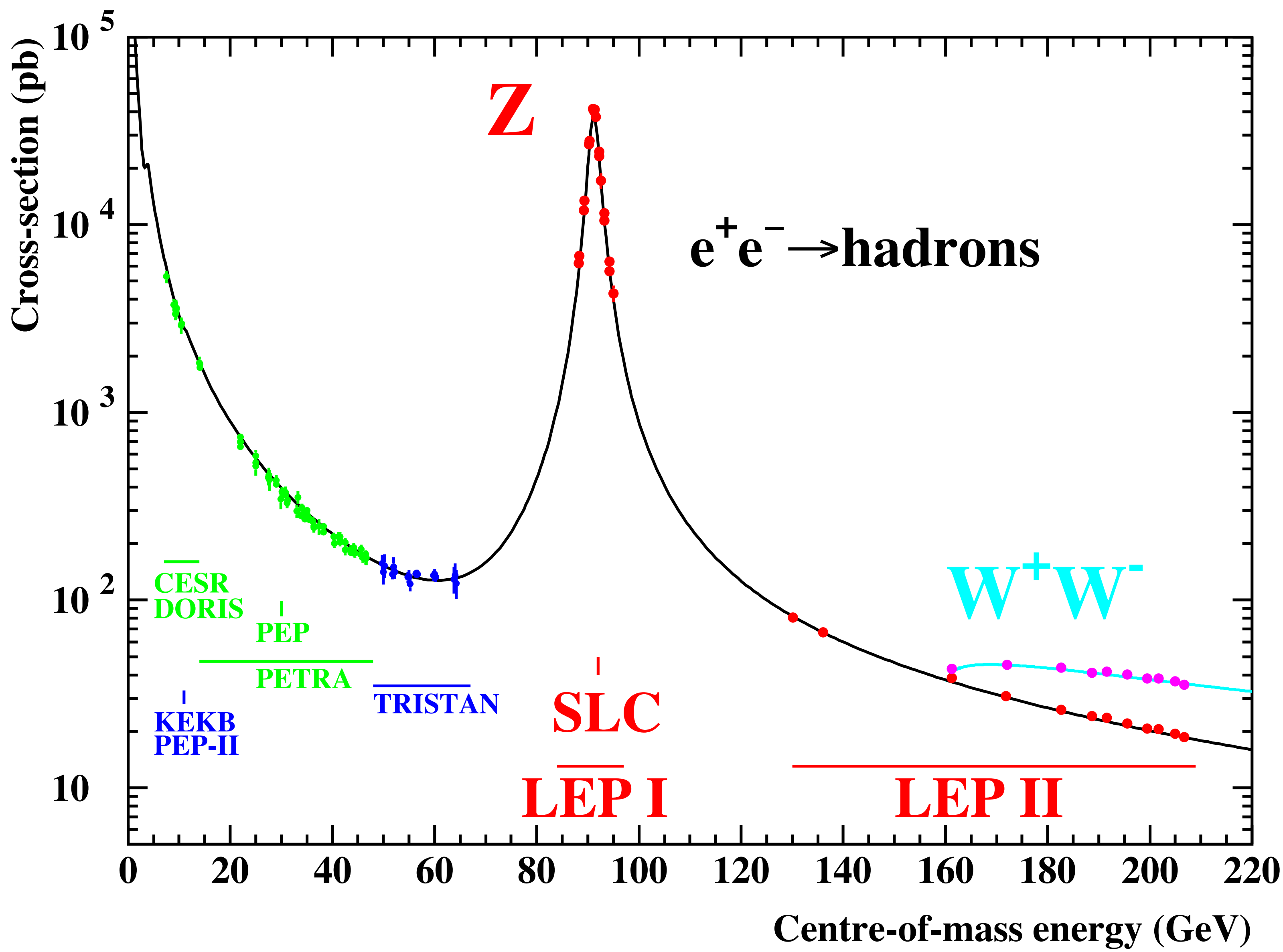
$$
\begin{aligned}
\mathcal{L} = \quad & -\frac{1}{4g_3^2}G_{\mu\nu}^2 - \frac{1}{4g_2^2}W_{\mu\nu}^2 - \frac{1}{4g_Y^2}B_{\mu\nu}^2 + |D_\mu H|^2 + V(H) \\
& \bar{q}_L \,\not{D} q_L + \bar{u}_R \,\not{D} u_R + \bar{d}_R \,\not{D} d_R + \bar{\ell}_L \,\not{D} \ell_L + \bar{e}_R \,\not{D} e_R \\
& + Y_u^{ij} \bar{q}_L^i H^\dagger u_R + Y_d^{ij} \bar{q}_L^i H d_R^j + Y_e^{ij} \bar{\ell}_L^i H e_R + \frac{\lambda^{ij}}{M}(H\ell^i)(H\ell^j) + \cdot \\
& + \sqrt{g} M_P^2 \left(R(g) - \lambda + \cdots\right)
\end{aligned}
$$

Status of particle physics





- Accelerator experiments had confirmed Standard Model again and again. Different experiments had verified each others findings
- All predicted particles have been found $W^{\pm}, Z^0$. t-quark. Higgs boson
- No new particles appeared so far!


Cross-section (pb)
10^5
10^4
10^3
10^2
10
Z
$e^+e^-\rightarrow$hadrons
CESR
DORIS
PEP
PETRA
KEKB
PEP-II
TRISTAN
SLC
LEP I
LEP II
W^+W^-
0
20
40
60
80
100
120
140
160
180
200
220
Centre-of-mass energy (GeV)

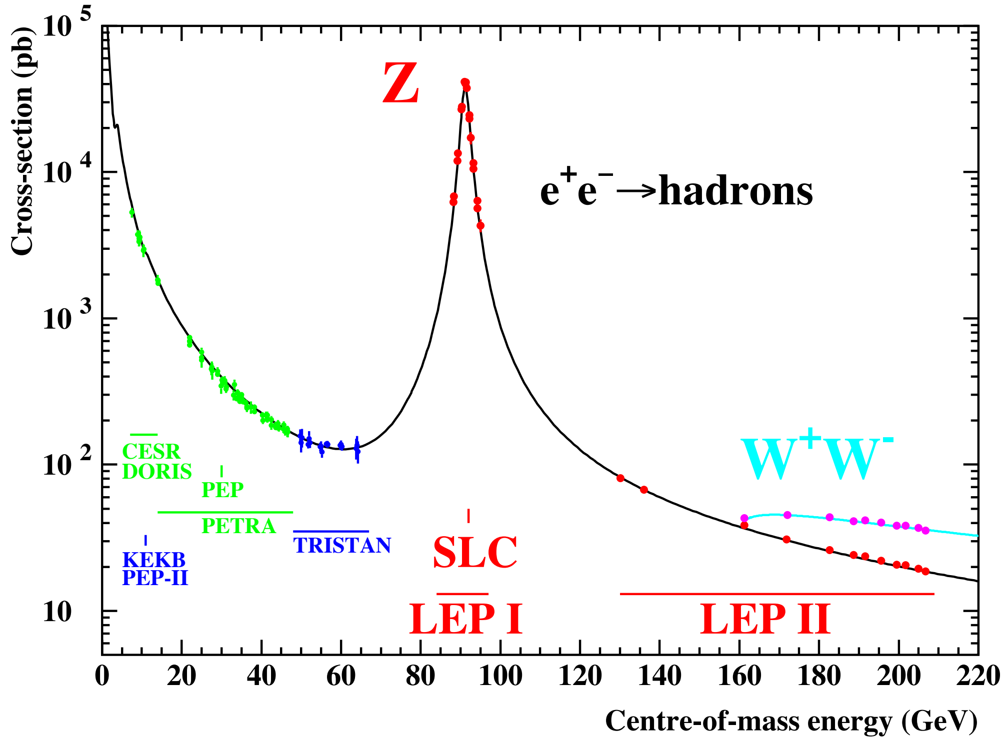


Figure 1: The cross-section for the production of hadrons in e^+e^- annihilations. The measurements are shown as dots with error bars. The solid line shows the prediction of the SM.

Analysing the resonant Z lineshape in the various Z decay modes leads to the determination of mass, total and partial decay widths of the Z boson as parametrised by a relativistic Breit-Wigner with an s dependent total width, m_Z, Γ_Z and $\Gamma_{f\bar{f}}$. Owing to the precise determination of the LEP beam energy, mass and total width of the Z resonance are now known at the MeV level; the combination of all results yields:

$$m_Z = 91.1875 \pm 0.0021 \text{ GeV} \quad (1)$$
$$\Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV}\,. \quad (2)$$

Not enough to generically ask ***why***

In order to infere ***where**** do we expect new physics to show up

We need to better understand ***what*** is the Standard Model

** at what energy scale*

Effective field theory approach to particle physics

working at tree level first

Physical scales & couplings

Ex: most general Lagrangian for scalar field

$$\mathcal{L} = \partial_\mu\phi\partial^\mu\phi - m^2\phi^2 + \lambda_4\phi^4 + \frac{\lambda_6}{\Lambda^2}\phi^6 + \frac{\lambda_8}{\Lambda^4}\phi^8 + \cdots$$
$$+\frac{\eta_4}{\Lambda^2}\phi^2\partial_\mu\phi\partial^\mu\phi + \frac{\eta_6}{\Lambda^4}\phi^4\partial_\mu\phi\partial^\mu\phi\cdots$$

dimensions ➡

$$\begin{cases} [\mathcal{L}] = \dfrac{Energy}{(Length)^3} = E^4 \\ [\partial_\mu] = \dfrac{1}{Length} = E \end{cases}$$

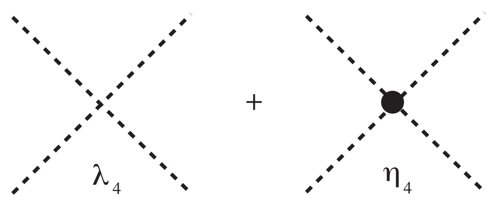
$$[\phi] = E$$

$\lambda_i,\ \eta_i\ =$ dimensionless

(assume $O(1)$)

assuming $m \lesssim E \ll \Lambda$

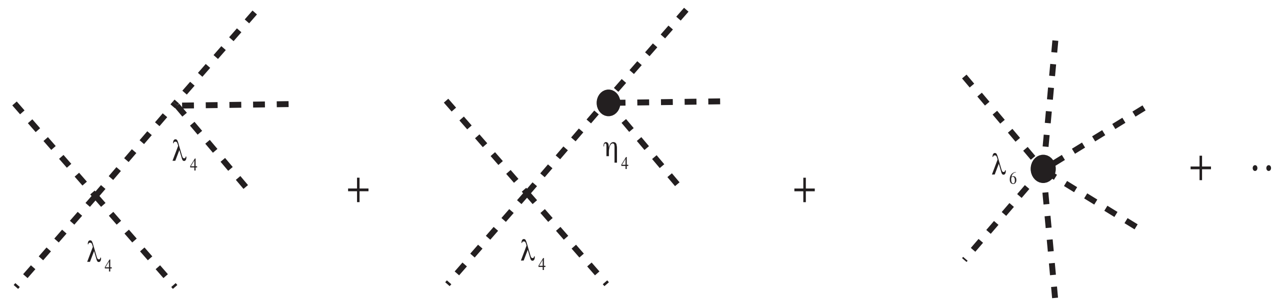
$$\mathcal{A}(2 \to 2) =$$



$$= \lambda_4 + \eta_4\frac{E^2}{\Lambda^2} \xrightarrow{E\to 0} \lambda_4$$

$$\mathcal{L} = \partial_\mu \phi \partial^\mu \phi - m^2 \phi^2 + \lambda_4 \phi^4 + \frac{\lambda_6}{\Lambda^2} \phi^6 + \frac{\lambda_8}{\Lambda^4} \phi^8 + \cdots$$
$$+ \frac{\eta_4}{\Lambda^2} \phi^2 \partial_\mu \phi \partial^\mu \phi + \frac{\eta_6}{\Lambda^4} \phi^4 \partial_\mu \phi \partial^\mu \phi \cdots$$

$\mathcal{A}(2 \to 4) =$



$$\frac{1}{E^2} \left\{ \quad \lambda_4^2 \quad + \quad \lambda_4 \eta_4 \frac{E^2}{\Lambda^2} \quad + \quad \lambda_6 \frac{E^2}{\Lambda^2} \right.$$

$E \ll \Lambda$ only a finite number of terms in the lagrangian are important

the infinite set of couplings with negative mass dimensions is *irrelevant* coupling g with dimension $[g] = d$

$$\bar{g} \equiv g E^{-d}$$

dimensionless quantity controlling strength of interaction

weak coupling $\longleftrightarrow$ $\bar{g} \ll 1$

★ d > 0 : relevant at small E

- *Ex: can treat mass as perturbation at E>> m* $\left(\bar{m}^2 = \frac{m^2}{E^2}\right)$

★ d = 0 : relevant at all energies (marginal)

- *gauge and Yukawa couplings*

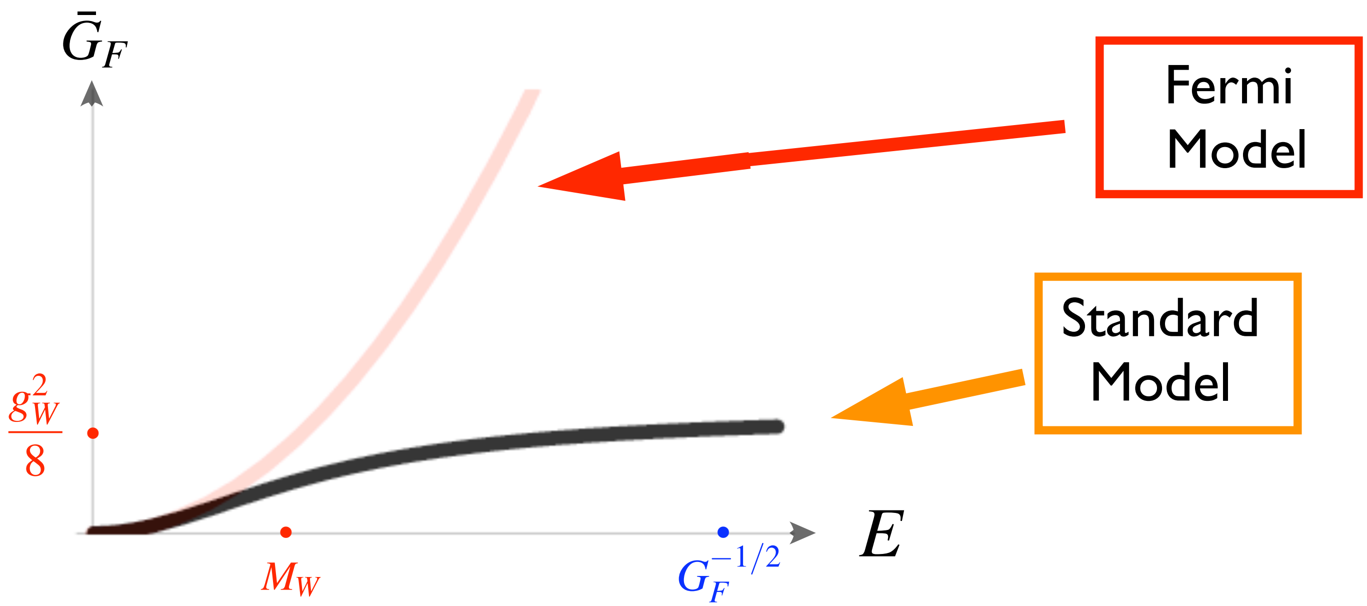
★ d < 0 : irrelevant al small E

- *perturbative expansion breaks down at high enough E*

Ex. : Fermi Lagrangian

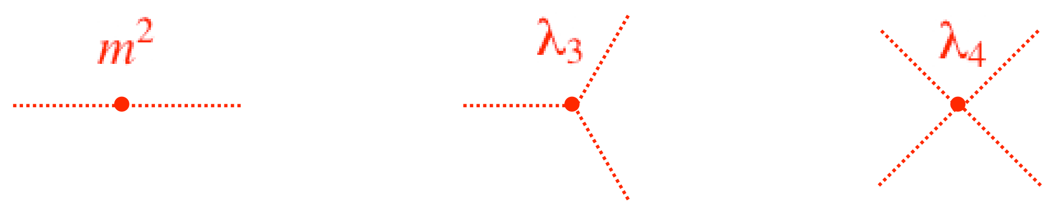
$$L_{Fermi} = G_F \left(\bar{p}\gamma_\mu n\right)\left(\bar{e}\gamma^\mu \nu\right)$$

$$\bar{G}_F \equiv G_F E^2$$



Imagine all couplings with $d_i < 0$ scale like inverse powers of a single scale Λ

dynamics at $E << \Lambda$ ⟷ couplings with $d_i \geq 0$



- $(m^2, \lambda_3, \lambda_4)$ fully describe an elementary (pointlike) particle
- $(\lambda_5, \lambda_6, \ldots)$ correspond to inner structure
- to probe structure, $E \approx \Lambda$ is needed ➡ wavelength $\approx \dfrac{1}{\Lambda}$

Now at the quantum level......

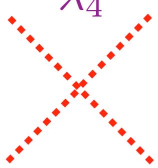
(a more physical picture of renormalizability)

Problem: internal momentum of loops is not fixed by external momentum

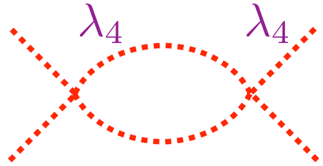
→ contributions enhanced by powers of cut-off

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda_4\phi^4 - \frac{1}{6!}\frac{\lambda_6}{\Lambda^2}\phi^6 - \frac{\lambda_8}{\Lambda^4}\phi^8 + \cdots$$

$\mathcal{A}(2 \to 2) =$

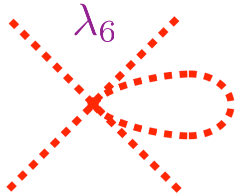


+



$$\Rightarrow \lambda_4\left(1 + \frac{\lambda_4}{32\pi^2}\ln(-\frac{s}{\Lambda^2}) + \ldots\right)$$

+



$$\Rightarrow \frac{1}{\Lambda^2}\frac{\lambda_6}{32\pi^2}\int_0^{\sim\Lambda^2}\frac{p^2dp^2}{p^2+m^2} \to \frac{\lambda_6}{32\pi^2}$$

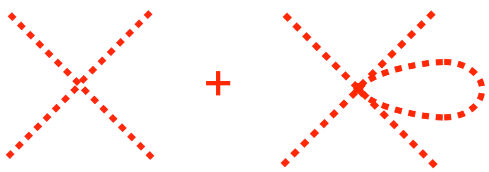
does not vanish when $\Lambda \to \infty$

tree level $\frac{\lambda_6 E^2_{external}}{\Lambda^2}$ → small

loop level $\frac{\lambda_6 E^2_{virtual}}{\Lambda^2}$ → in principle $E_{virtual} \sim \Lambda$

not small

- ❖ Apparently operators of arbitrarily high dimension matter!
- ❖ But notice that UV enhanced contribution is ***local***



$$\lambda_4' \equiv \lambda_4 + \frac{\lambda_6}{32\pi^2}$$

UV enhanced contribution is just a renormalization of quartic term

Result generalizes to all orders

Accidental symmetries

Accidental symmetries

$$E \ll \Lambda$$

dynamics determined by a ***few*** `renormalizable' couplings

$$\Downarrow$$

extra (accidental) symmetries

Example: parity in QED is respected by `renormalizable' interactions

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}i\gamma^{\mu}D_{\mu}\psi + \bar{\psi}(m_1 + i\gamma_5 m_2)\psi + \frac{a}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$(m_1 + i\gamma_5 m_2) \quad \to \quad m = \sqrt{m_1^2 + m_2^2}$ **by chiral rotation** $\psi \quad \longrightarrow \quad e^{i\beta\gamma_5}\psi$

$F_{\mu\nu}\tilde{F}^{\mu\nu}$ **= total derivative**

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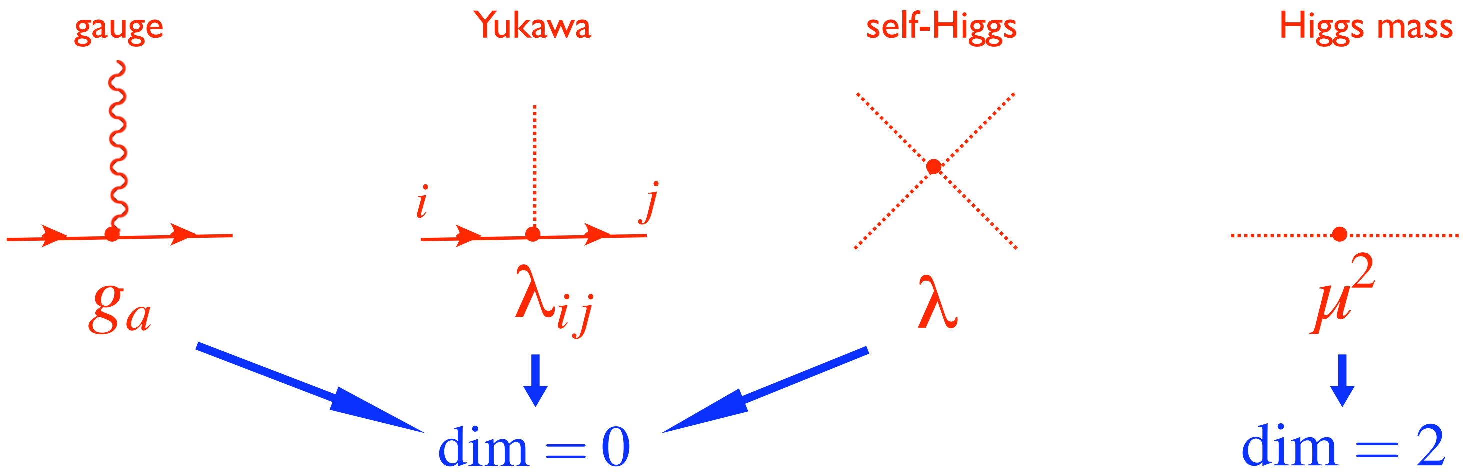
dim 6 operator violates parity

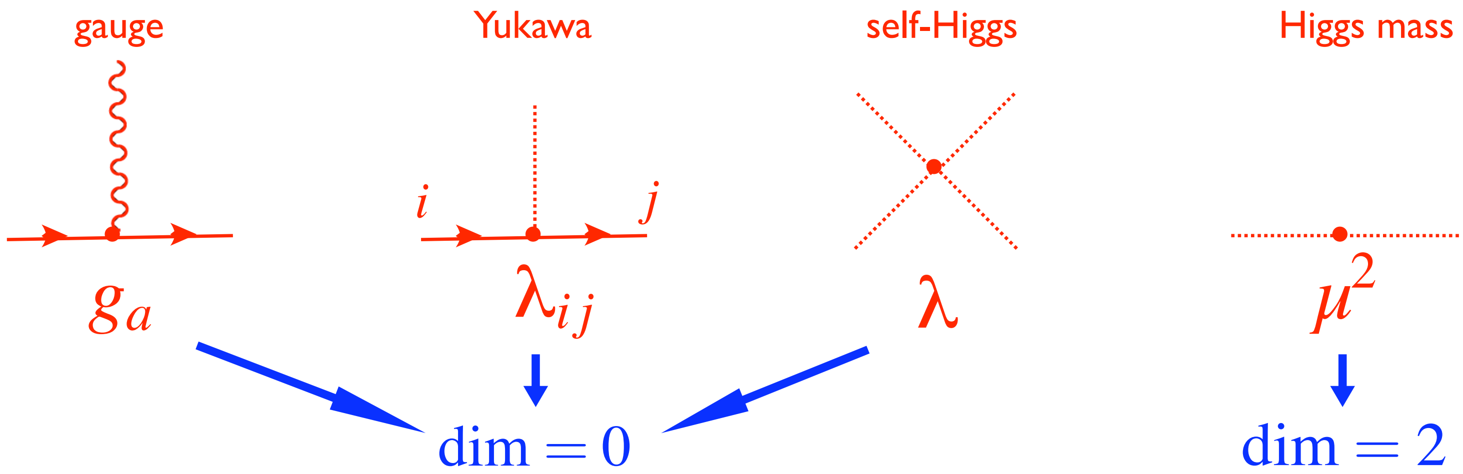
$$O_{\not{P}} = \frac{1}{\Lambda^2}(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma_\mu\gamma_5\psi)$$

generated in SM by Z-exchange $\frac{1}{\Lambda^2} \sim G_F = \frac{1}{v^2}$

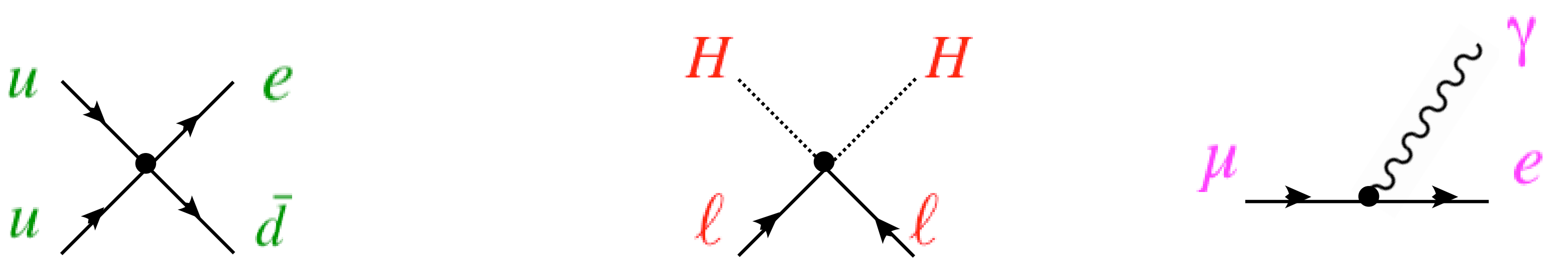
Standard Model interactions



Standard Model interactions



★ By allowing dim < 0 we would also have:



$$\frac{1}{\Lambda_{\not B}^2}\left(u_\alpha C\gamma_\mu u_\beta\right)\left(e C\gamma^\mu d_\delta\right)\varepsilon^{\alpha\beta\gamma}$$

$$\frac{1}{\Lambda_{\not L}}\left(\ell^a C\,\ell^b\right)H_a H_b$$

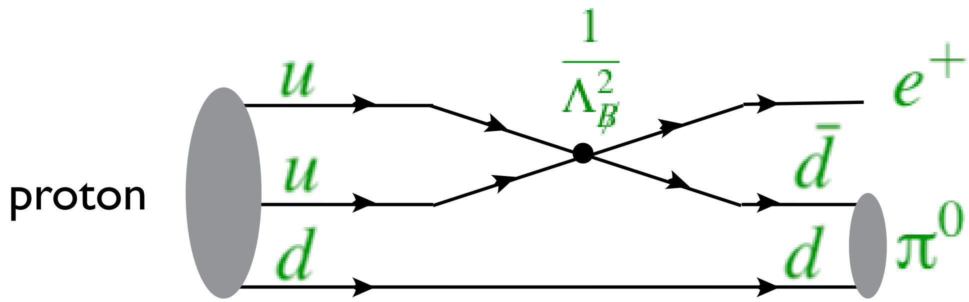
$$\frac{m_\mu}{\Lambda_{\not F}^2}\left(\bar{e}\,\gamma_\mu\gamma_\nu\,\mu\right)F^{\mu\nu}$$

Baryon number violation

Lepton number violation

Flavor violation

1) B+L violation: proton decay



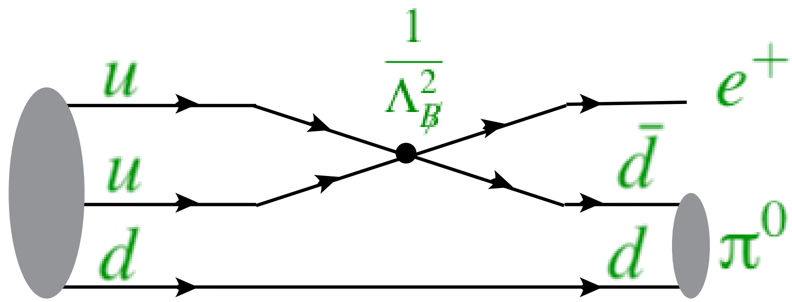
$$p \to e^+ \pi^0$$

Superkamiokande: $\tau_p > 8.2 \times 10^{33}$ years

$$\Rightarrow \quad \Lambda_{\not B} \geq 10^{15}\,\mathrm{G}eV$$

1) B+L violation: proton decay

proton



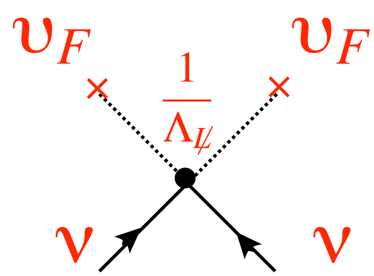
$$p \longrightarrow e^+ \pi^0$$

Superkamiokande: $\tau_p > 8.2 \times 10^{33}$ years

$$\Lambda_{\not B} \geq 10^{15} \mathrm{G}eV$$

2) L violation: neutrino masses

$$H \longrightarrow \upsilon_F$$



$$m_\nu \sim \frac{\upsilon_F^2}{\Lambda_{\not L}}$$

observed neutrino oscillations: $m_\nu \sim 0.1 \mathrm{e}V$ $\Lambda_{\not L} \sim 10^{14} \mathrm{G}eV$

3) Flavor violation

$$\mathcal{L} = \bar{q}_L \hat{Y}_d H^\dagger d_R + \bar{q}_L V_{CKM} \hat{Y}_u H u_R + \bar{\ell} \hat{Y}_\ell H^\dagger e_R$$

$$\hat{Y}_d = \begin{pmatrix} \lambda_d & & \\ & \lambda_s & \\ & & \lambda_b \end{pmatrix} \qquad \hat{Y}_u = \begin{pmatrix} \lambda_u & & \\ & \lambda_c & \\ & & \lambda_t \end{pmatrix} \qquad \hat{Y}_\ell = \begin{pmatrix} \lambda_e & & \\ & \lambda_\mu & \\ & & \lambda_\tau \end{pmatrix}$$

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- absence of ν_R $\Rightarrow$ L_e, L_μ, L_τ are conserved

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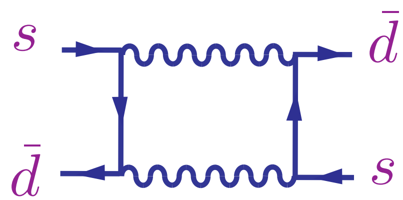
- absence of ν_R ⇒ L_e, L_μ, L_τ are conserved
- very special quark Flavor violation all due to V_{CKM} ⇒ Glashow-Iliopoulos-Maiani (GIM) suppression mechanism

$K - \bar{K}$ mixing



$$\sim \frac{G_F \alpha_W}{4\pi} (\sin\theta_C \cos\theta_C)^2 \left(\frac{m_c}{M_W}\right)^2 \left[\bar{d}_L \gamma^\mu s_L\right]^2$$

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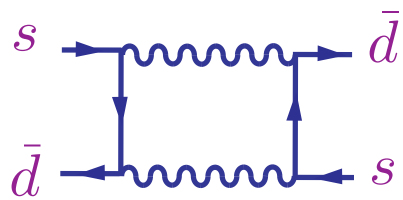
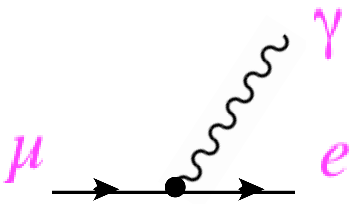
$K - \bar{K}$ mixing



$$\sim \frac{G_F \alpha_W}{4\pi} (\sin\theta_C \cos\theta_C)^2 \left(\frac{m_c}{M_W}\right)^2 \left[\bar{d}_L \gamma^\mu s_L\right]^2$$

non-renormalizable contribution $\quad \frac{1}{\Lambda_{\not{F}}^2}\left[\bar{d}_L \gamma^\mu s_L\right]^2 \qquad \left.\frac{\Delta m_K}{m_K}\right|_{\exp} \longrightarrow \Lambda_{\not{F}} > 10^6\,\mathrm{GeV}$

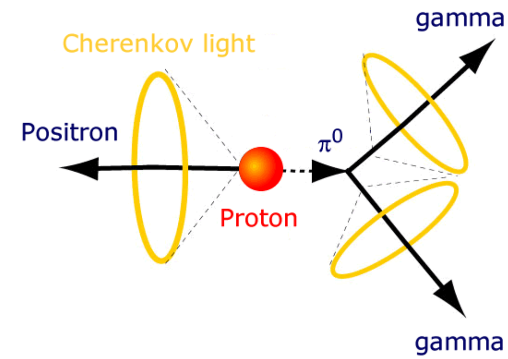
lepton flavor violation



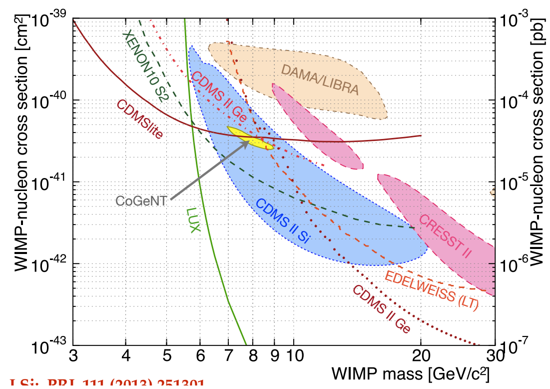
$$\frac{m_\mu}{\Lambda^2_{\not F}} \,(\bar{e}\,\gamma_\mu \gamma_\nu\, \mu)\, F^{\mu\nu}$$

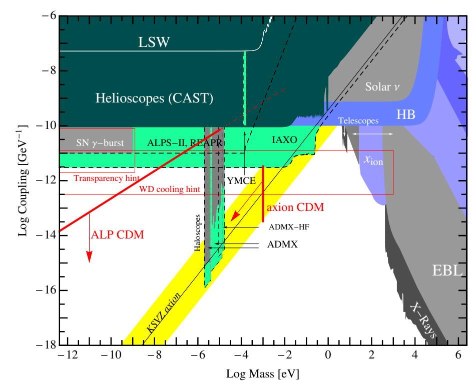
$\mathrm{Br}(\mu \to e\gamma) < 10^{-11}$ ➡ $\Lambda_{\not F} > 10^6\,\mathrm{GeV}$

No new physics (Exotics)

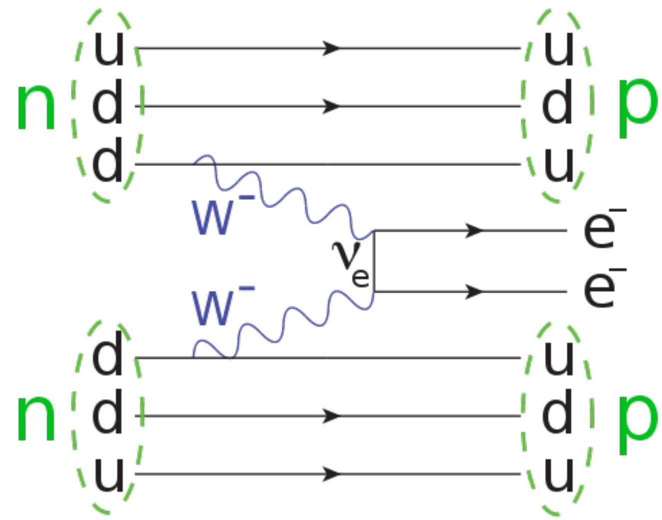


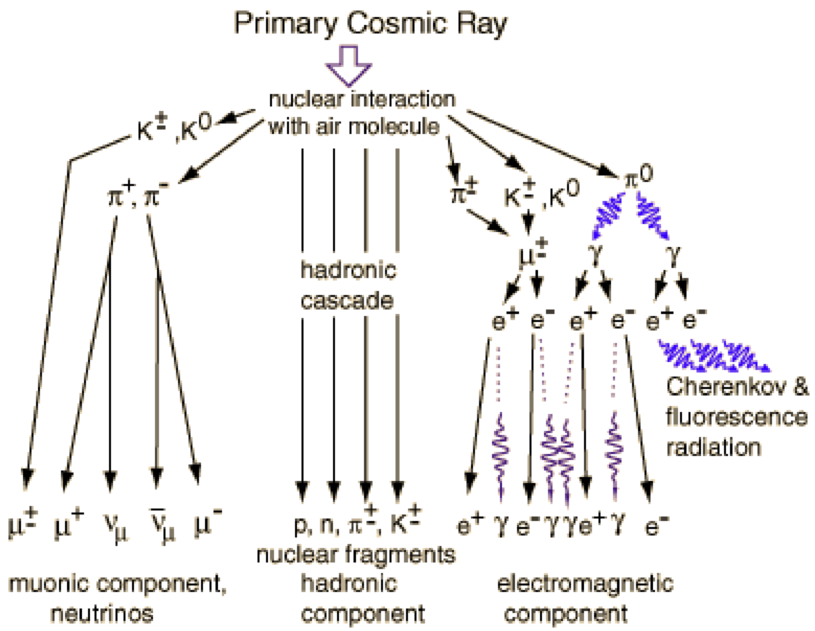
- × Proton decay: $\tau_{p\to\pi^0+e^+} > 8.2\times10^{33}$ years
  baryon number violation
- × New weakly interacting massive particles

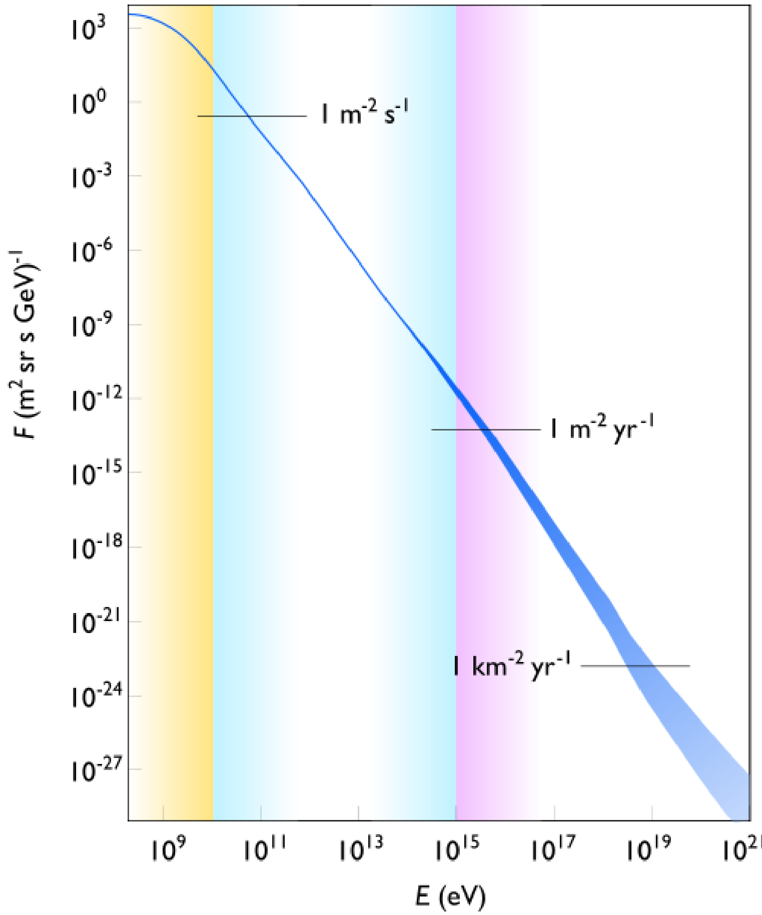




- × Axion searches
- × Millicharges
- × Paraphotons
- × Neutron electric dipole moment
  CP violation in strong interactions
- × Neutrinoless double beta decay
  Lepton number violation
- × No $\mu\to e+\gamma$ or $\mu^+\to e^+e^-e^+$







- Historically cosmic rays were the first accelerators (positron, muons, . . .)

- Today we detect photons, electrons/positrons, protons/antiprotons, nuclei (iron), neutrinos up to very high energies

- Everything is consistent with our knowledge of astrophysics and particle physics

Structure of the Standard Model
Why it is the way it is

Structure of the Standard Model

Gauge Group → $G = SU(3) \times SU(2) \times U(1)_Y$

matter fermions →

$$\left\{ \begin{array}{lclcccc} q_L & = & (3 & , & 2 & , & 1/3) \\ u_R & = & (3 & , & 1 & , & 4/3) \\ d_R & = & (3 & , & 1 & , & -2/3) \\ l_L & = & (1 & , & 2 & , & -1) \\ e_R & = & (1 & , & 1 & , & -2) \end{array} \right.$$

- why this apparently bizarre spectrum ?
- why is hypercharge quantized ?

non-abelian group
Ex: SU(2) →

$$[T_3, T_\pm] = \pm T_\pm$$
$$[T_+, T_-] = 2T_3$$

⇒ $T_3|\psi\rangle = \frac{n}{2}|\psi\rangle$ (integer ↓ n)

abelian group: no quantization condition

Can one build new theory with non-abelian hypercharge ?

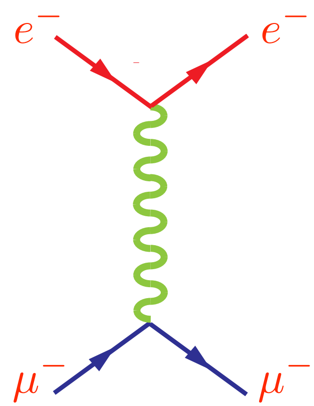
General Relativity at the quantum level only makes sense as an ***Effective*** Quantum Field Theory

There is an absolute upper bound on the energy scale at which General Relativity makes sense

Gravity couples to all other particles

absolute upper bound on energy scale up to which the SM can be valid

$\mathcal{A}_{\text{gravity}} =$



$= \frac{s}{M_P^2}\frac{1}{t}$

quantum effects untractable at $E \sim M_P \simeq 10^{19}\ \text{GeV}$

M_P is huge and thus gravity is not necessarily of urgent concern for the LHC

But previous argument only sets an upper bound on relevant gravity scale. In the scenario of large extra dimensions gravity becomes indeed strong at around a TeV

The fate of gravity is of crucial importance to develop a theory of the very early universe

Strength of forces at E $\approx M_Z$

SU(3)	→	$g_3^2 \simeq 1.5$
SU(2)	→	$g_W^2 \simeq 0.42$
$U(1)_Y$	→	$g_Y^2 \simeq 0.13$

- they differ, but n o t w i l d l y
- strength of gravity at E $\approx M_Z$

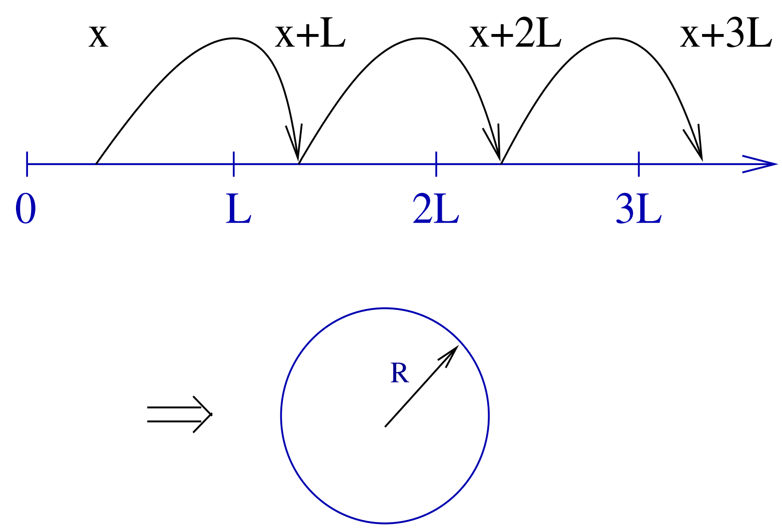
$$G_N M_Z^2 \equiv \frac{M_Z^2}{M_P^2} \sim 10^{-34}$$

Extra dimensions. Domain wall

- Old idea (1920s, Kaluza & Klein): Unify gravitation and electromagnetism in a 5D gravity theory. 5D Gravity where 5th direction is a **circle** gives 4D Gravity + Electromagnetism

Extra spatial dimensions with points periodically identified

1 Extra Dimension: equivalent to a circle



with $R = L/2\pi$. We identified the points

$$x \sim x + L \sim x + 2L \sim x + 3L \sim \cdots$$

Compactification

- When field propagates in one extra dimension

$$P_M = P_\mu + P_5$$

with $\mu = 0, 1, 2, 3,\ M = \mu, 5$.
- But XD is compact $\Rightarrow$ P_5 is quantized:
periodicity $\Rightarrow$ wavelength has to be integer number of $2\pi R$.

$$P_5 = \frac{n}{R}\ , \qquad\qquad (n = 0, 1, 2, 3, \cdots)$$

Compactification

- If field has mass M

$$P_M P^M = P_\mu P^\mu - P_5^2 = P_\mu P^\mu - \frac{n^2}{R^2}$$

- From the 4D point of view:

$$P_\mu P^\mu = M^2 + \frac{n^2}{R^2}$$

- E.g. for a photon (or graviton) $M = 0$.
  There is a "$n = 0$-mode" with zero mass (our photon/graviton), plus infinite excitations with masses n/R.

Universal Extra Dimensions

For example, a scalar field $\Phi(x, y)$ in one extra dimension:

$$S[\Phi(x,y)] = \frac{1}{2}\int d^4x\, dy \left(\partial_M \Phi \partial^M \Phi - M^2 \Phi^2\right)$$

- Periodic boundary conditions:

$$\Phi(y) = \Phi(y + 2\pi R)$$

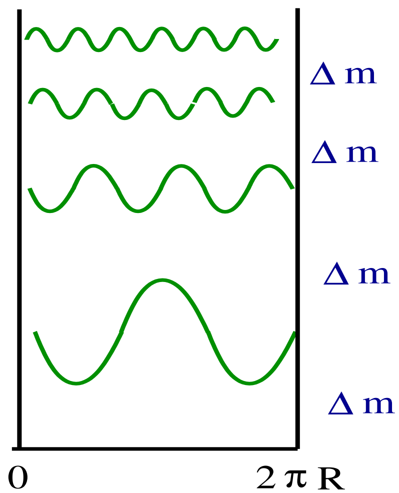
- Expand in Fourier modes:

$$\Phi(x,y) = \frac{1}{\sqrt{\pi R}} \sum_{n=0} \left[\phi_n(x) \cos\left(\frac{ny}{R}\right) + \tilde{\phi}_n(x) \sin\left(\frac{ny}{R}\right)\right]$$

- $\phi_n(x)$ and $\tilde{\phi}_n(x)$ are 4D fields.

Compact Extra Dimensions - Spectrum

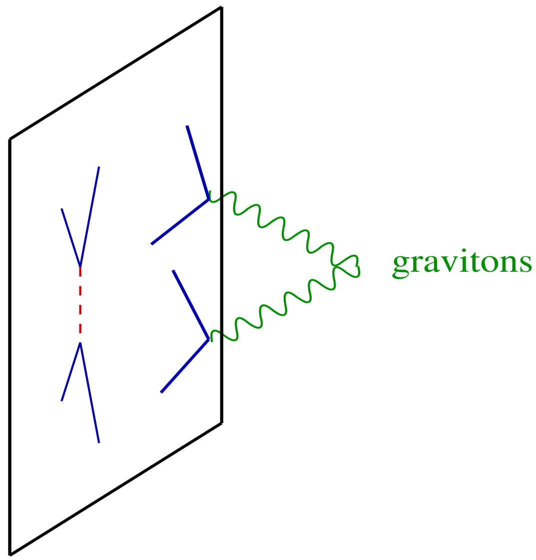
Compact extra dimensions $\Rightarrow$ particle excitations (Kaluza-Klein tower)



Mass gap $\Delta m \sim 1/R$

Large Extra Dimensions

- Assume space has $3 + n$ dimensions.
- The extra n dimensions are compact and with radius R.
- All particles are confined to a 3-dimensional slice ("brane").
- Gravity propagates in *all* $3 + n$ dimensions.



Large Extra Dimensions (Arkhani-Hamed, Dimopoulos, Dvali '98)

- Gravity appears weak ($M_P \ll M_W$), because it propagates in large extra dimensions... Its strength is diluted by the volume of the n extra dimensions.
- Fundamental scale is $M_* \sim M_W$, not M_P

$$M_P^2 \sim M_*^{n+2} R^n$$

- There is no hierarchy problem:
  The fundamental scale of Gravity

$$M_* \sim 1 \text{ TeV}$$

Large Extra Dimensions

If we require $M_* = 1$ TeV:

$$R \sim 2 \cdot 10^{-17}\, 10^{\frac{32}{n}}\,\mathrm{cm}$$

- $n = 1 \Longrightarrow R = 10^8$ Km. Already excluded!
- $n = 2 \Longrightarrow R \simeq 2$ mm. Barely allowed by current gravity experiments.
- $n > 2 \Longrightarrow R < 10^{-6}$ mm. This is fine.

Large Extra Dimensions

E.g. for

$$n = 2 \longrightarrow \Delta m = 10^{-3} \text{ eV}.$$
$$n = 3 \longrightarrow \Delta m = 100 \text{ eV}.$$
$$\vdots$$
$$n = 7 \longrightarrow \Delta m = 100 \text{ MeV}.$$

Why fermions in the Standard Model are chiral?

Why fermions are chiral?

Extra dimensions can be a potential explanation why fermions of the Standard Model are chiral

- Recall from the previous lecture: Landau levels in the magnetic field

$$E_n^2 - p_z^2 = eB(2n+1) + 2s_z eB \tag{1}$$

- Spectrum has three quantum numbers:

  ▷ $n = 0, 1, 2 \ldots$
  ▷ $-\infty \leq p_z \leq +\infty$
  ▷ $s_z = \pm\frac{1}{2}$

- Consider $n = 0$. For $s_z = -\frac{1}{2}$ the spectrum (6) becomes

$$E^2 = p_z^2 \qquad \text{massless 1-dimensional fermion} \tag{2}$$

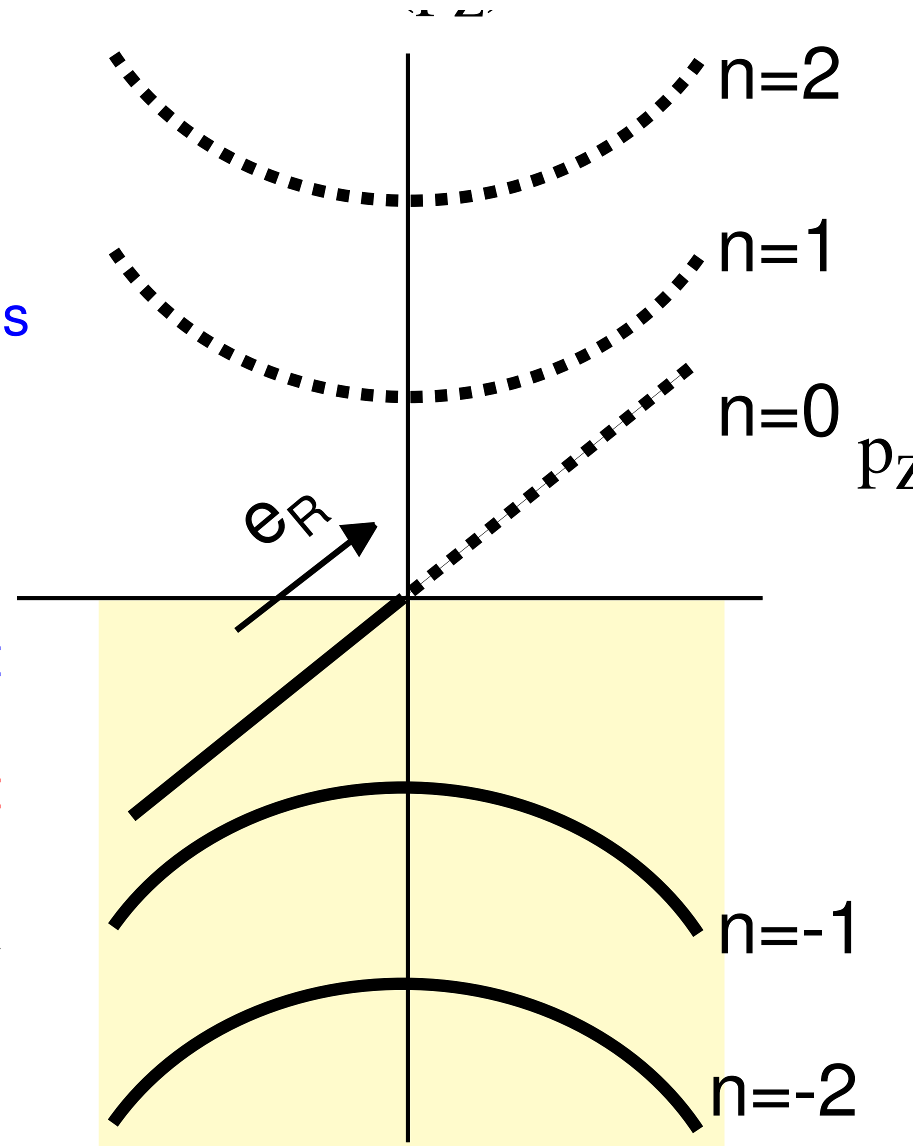
for $s_z = +\frac{1}{2}$ there is **no** massless mode

- Particles with $\vec{B} \cdot \vec{s} < 0$ have **massless** branches:

$$E = \begin{cases} -p_z & \text{move down along z-axis} \\ p_z & \text{move up along z-axis} \end{cases}$$

- Dirac vacuum $\leftrightarrow$ all states $E < 0$ are filled:
  - $E = -p_z < 0 \Rightarrow \vec{p} \cdot \vec{s} < 0$ – left particles
  - $E = p_z < 0 \Rightarrow \vec{p} \cdot \vec{s} > 0$ – right particles

- **Magnetic field** had broken parity and created chiral 1+1 dimensional modes

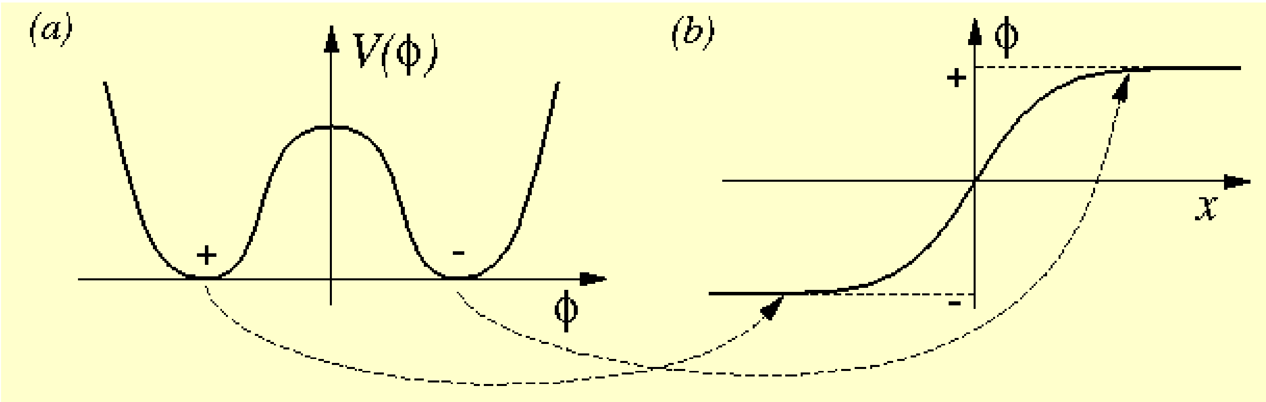


- Real scalar field:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{\lambda}{4}(\phi^2 - v^2)^2$$

- two vacua: $\phi = +v$ and $\phi = -v$

- Is there a solution such that $\phi(z \to -\infty) = -v$ and $\phi(z \to -\infty) = +v$?



– **KINK solution**

- What so special about it? Its energy is **finite**:

$$E[\phi] = \int dx \left[\frac{1}{2} (\nabla \phi)^2 + V(\phi) \right]$$

— finite because $\phi(\pm\infty) = \pm v$

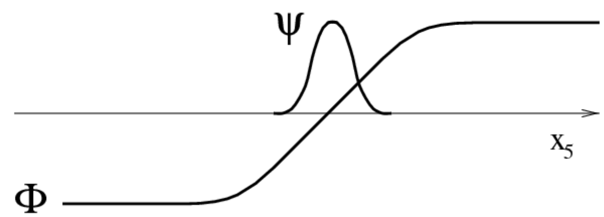
- To change this solution into the vacuum one (for example, into $\phi = -v$) we need to deform $\phi(+\infty)$ from $+v$ to $-v$.

- However, any solution with $\phi(+\infty) \neq \pm v$ will automatically have **infinite energy** ($V(\phi) \neq 0$ at infinity)

- $\Rightarrow$ kink is **topologically stable**

- Now add fermions:

$$\mathcal{L}_\Psi = \bar{\Psi} \not{\partial} \Psi + g \bar{\Psi} \phi \Psi$$



Chiral fermions on the domain wall

- Idea: make kink-like solution in 5th dimension. Our world is 3+1 dimensional domain wall in the 4+1 dimensional space
- **Non-compact extra dimensions!**
- Fermions are massive for $x_5 \to \pm\infty$ and massless for $x_5 = 0$
- Increasing coupling g and vev v you can make fermions arbitrarily massive away from the domain wall
- Dirac equation (keep in mind that $i\gamma_5$ is just a Dirac matrix for the 5th component)

$$i\gamma^\mu \partial_\mu \psi(x) + \gamma_5 \psi(x) \frac{f'(x_5)}{f(x_5)} = -g\phi(x_5)\psi(x)$$

where $f(x_5)$ is the profile of the fermion mode in 5th dimension

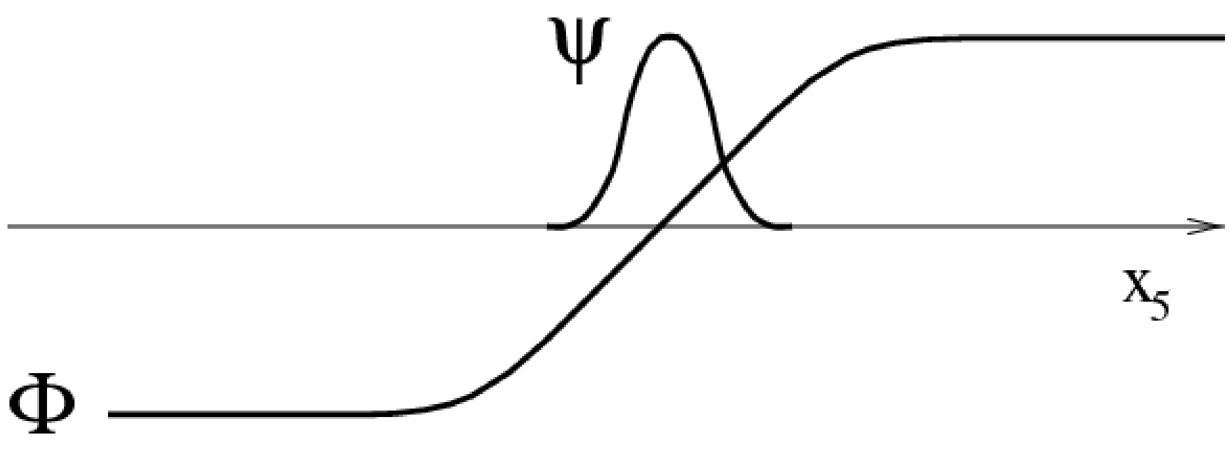
Chiral fermions on the domain wall

- Let us choose $\gamma_5\psi = \pm\psi$. Then for large x_5 we get:

$$\frac{f'(x_5)}{f(x_5)} = \mp g v \qquad (3)$$

- For one chirality the solution is normalizable ($f(x_5) \sim e^{-gvx_5}$ for $x_5 \to \infty$. The other mode is exponentially growing in the bulk

- $\Rightarrow$ The fermion is **localized** on the domain wall and has definite chirality (the opposite one would be for **anti-**kink



Strong CP problem

- Consider the Lagrangian of QCD:

$$\mathcal{L} = -\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu\, a} + \sum_q \bar{q}\left(i\, \not{D} - \mathcal{M}_{CKM}\right) q + \frac{\theta_0}{32\pi^2}G^a_{\mu\nu}\tilde{G}^{\mu\nu\, a}.$$

- The CKM matrix is non-diagonal and contains CP-violating phases. We know about this because we observed CP-violations in kaon decays.

- Recall, that s-quark carries a quantum number (**strangeness**) conserved in strong interactions. Its electric charge is the same as electric charge of d-quark. Therefore, there are two neutral kaons: $|K_0\rangle \equiv |d\bar{s}\rangle$ and $|\bar{K}_0\rangle \equiv |\bar{d}s\rangle$. It was observed that decays of $K_0 \to \pi^- e^+ \nu_e$ and decays of $\bar{K}_0 \to \pi^+ e^- + \bar{\nu}_e$ produced different number of electrons and positrons as the level $\sim 10^{-3}$

- this means that the mass matrix $\mathcal{M}_{CKM}$ contains CP-violating phase, δ_{CKM}. In the simplest case this phase can be thought of as

$$\mathcal{L}_{\text{quarks}} = \bar{q}(i\, \not{D} - m e^{i\delta_{CKM}\gamma_5})q$$

Strong CP problem

- We can choose to call "quarks" objects that are $\tilde{q} = e^{\frac{i}{2}\delta_{CKM}\gamma_5}q$
- If we try to do such a change of variables we would get additional term $\delta_{CKM}G^a_{\mu\nu}\tilde{G}^{\mu\nu\, a}$ in the presence of gluon field

  This is the same **axial anomaly** that we have discussed previously. If axial symmetry would be exact, than such change of variables left the whole Lagrangian invariant and would change only the mass term. However, because of the anomaly in the axial symmetry, we get additional term in the Lagrangian.

- The term $\delta_{CKM}G^a_{\mu\nu}\tilde{G}^{\mu\nu\, a}$ should lead e.g. to the appearance of electric dipole moment of neutron (CP-violating observable)
- The experimentally observed **absence** of such a dipole moment (or any other CP violation in the strong sector) places the limit on $\theta = \theta_0 + \delta_{CKM} \lesssim 10^{-9}$.
- The smallness of θ poses the **strong CP problem** as it is not clear why δ_{CKM} and θ_0 which are *a priory* unrelated are equal to each other with such precision.

Axions as the solution of strong CP problem

Let us promote the constant θ_0 to an additional (pseudo)scalar field a

$$\mathcal{L}_a = \frac{1}{2}\partial^\mu a \partial_\mu a + \left(\frac{a}{f_a} + \delta_{\rm CKM}\right)\frac{1}{32\pi^2}G^a_{\mu\nu}\tilde{G}^{\mu\nu\,a},$$

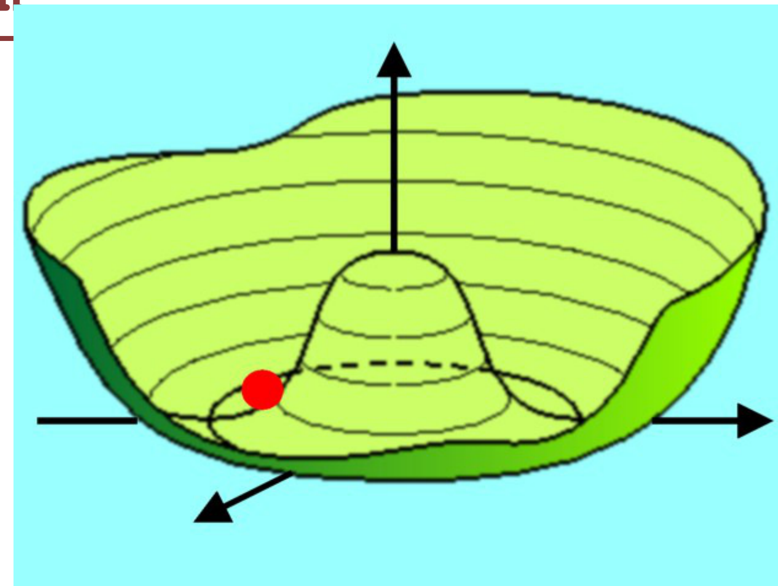
where a is the new scalar field (called **axion**) having additional **global** $U_{PQ}(1)$ symmetry (called **Peccei-Quinn symmetry**)

$$a \to a - f_a\delta_{CKM}$$

The strong CP problem thus becomes: "**where is the minimum of** a**?**"

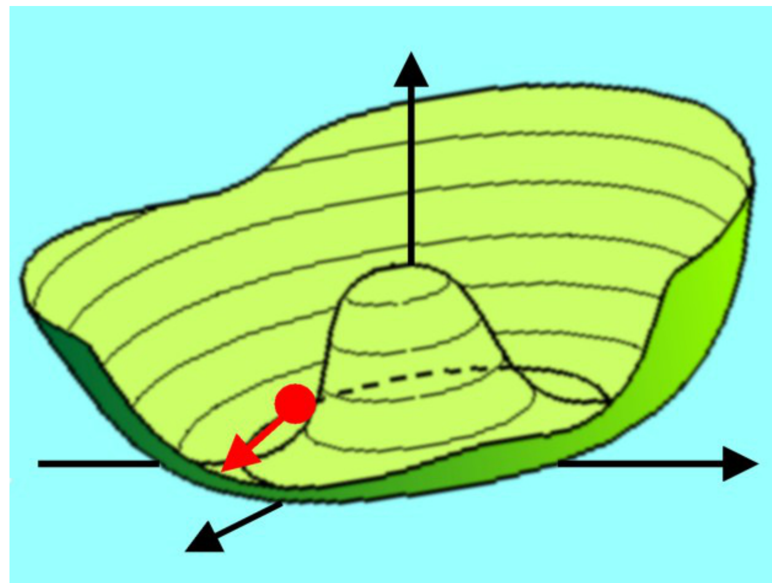
At $E \simeq \Lambda_{QCD}$, QCD non-perturbative effects generate the effective potential for field a. This potential is **periodic**. ($\frac{1}{32\pi^2}\int d^4x\, G^a_{\mu\nu}\tilde{G}^{\mu\nu\,a} = n$, $n \in \mathbb{Z}$, $a \to a + 2\pi f_a$ shifts the effective action by $e^{2\pi i n} = 1$)

Axions as the solution of strong CP problem



At $E \simeq f_a$

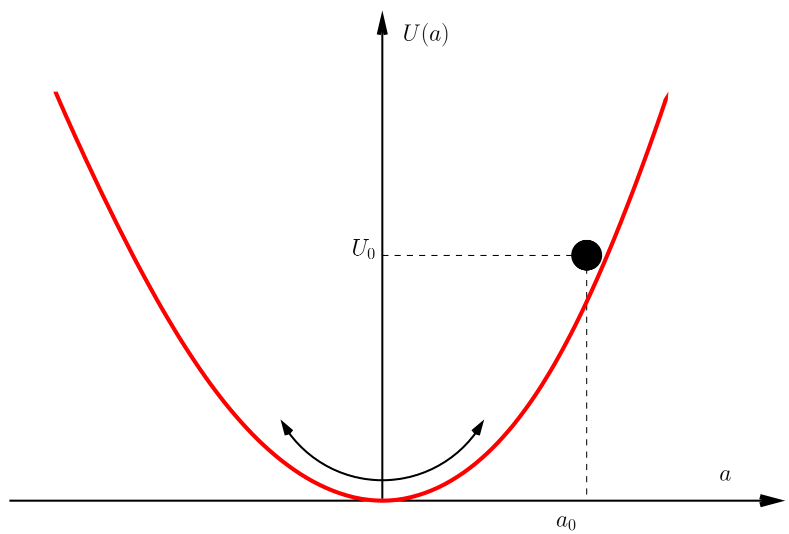
- $U_{PQ}(1)$ **spontaneously** broken;
- Axions settle in a Mexican hat.



At $E \simeq \Lambda_{QCD} \ll f_a$

- $U_{PQ}(1)$ **explicitly** broken by the QCD non-perturbative effects;
- Mexican hat tilts;
- Axions acquire a mass;
- CP symmetry is restored.

- At $T \gg \Lambda_{\rm QCD}$ potential $U(a) = 0$ and value of axion field plays no role (shift symmetry, only derivative matters).
- In the expanding Universe $\ddot{a} + 3H\dot{a} + \frac{\partial U}{\partial a} = 0$. $U(a) = 0 \Rightarrow$ the solution is $\dot{a} = 0$
- At some moment $T \approx \Lambda_{\rm QCD}$ field a acquires a potential (computations give $U(a) = \Lambda_{\rm QCD}^4(1 - \cos(a/f_a))$)
- Its initial value a_0 is generically off the minimum, so the field starts to oscillate



- The oscillations of the axion field around its minimum then leads to axion production.

- The presence of the "Hubble friction" damps the oscillations, finally axion settles at the minimum, providing a solution to the strong CP problem and creating a DM candidate

- Its energy density is $U(a_0) \sim \Lambda^4_{\text{QCD}}$.

- After oscillations stopped – number of axions does not change. Then the energy density *today* is

$$\rho_a \approx \frac{U(a_0)}{(1+z_{\text{QCD}})^3} \approx \Lambda_{\text{QCD}} T^3_{\mathrm{CMB}}$$

  Using $\Lambda_{\text{QCD}} \sim 100$ MeV and $T_{\mathrm{CMB}} = 2.7$ K we get $\rho_a \sim 0.1\rho_{crit}$ (**check!**) – correct DM abundance

Axions as DM particles

- Notice that axion particles are very **cold** (their momentum $p \sim H_0$ (characteristic variation at horizon scales) (**similar to generation of scale-free perturbations at the inflationary stage**)

- The effective mass of the axion is about

$$m_a \simeq 6 \times 10^{-6}\ \text{eV} \left(\frac{10^{12}\ \text{GeV}}{f_a}\right).$$

  $m_a \gg H_0$ – despite such a low mass, it constitutes **cold** DM

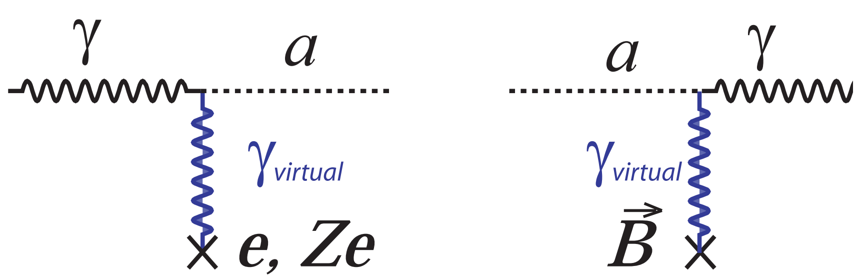
- Characteristic property : interaction with **photons**:

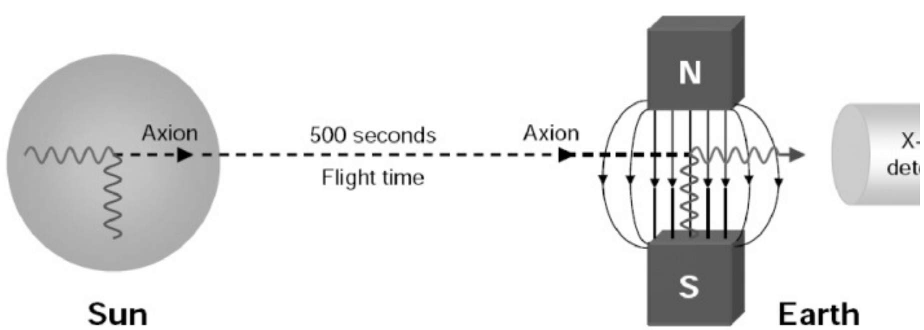
$$\mathcal{L}_a = \frac{a}{4f_a} F_{\mu\nu}\tilde{F}^{\mu\nu} = \frac{a}{f_a}\vec{E}\cdot\vec{H}$$

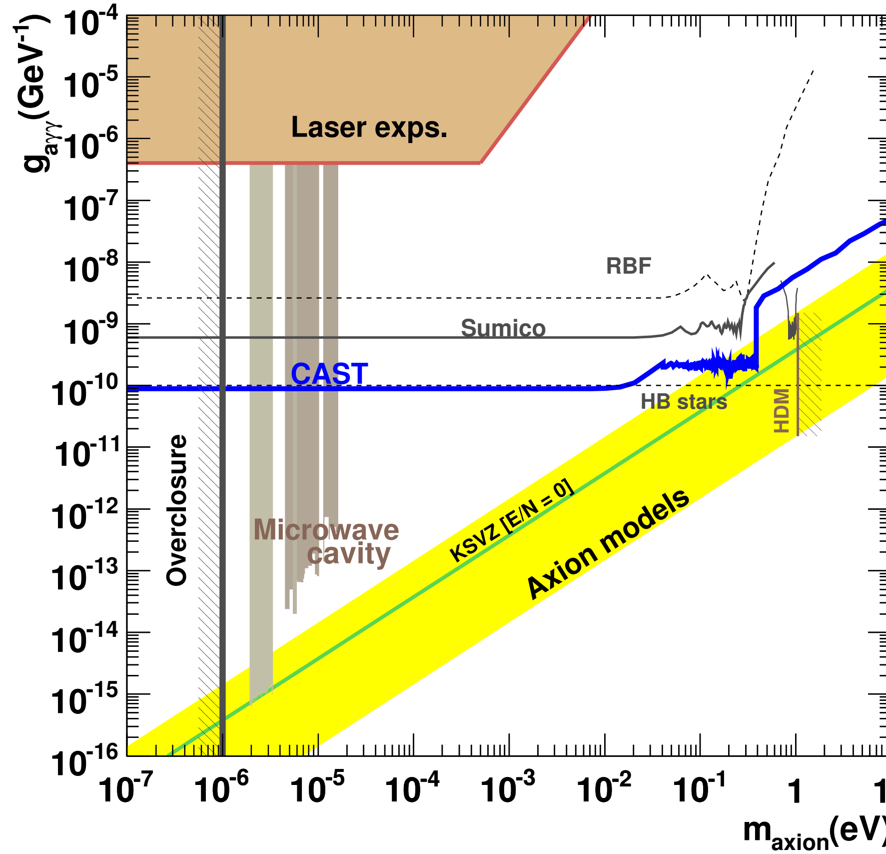
`http://wwwth.mpp.mpg.de/members/raffelt/pages/reviews.html`

Besides the non-relativistic dark matter axions, relativistic axions could be produced inside stars with the help of **Primakoff effect**, and then be captured by the **ground-based gelioscope**.

An example: **CERN Axion Solar Telescope (CAST)**.







Hierarchy problem

The hierarchy problem

$$\mathcal{L}_{SM} = \mathcal{L}^{d=2} + \mathcal{L}(g,\lambda)^{d=4} + \frac{1}{\Lambda}\mathcal{L}^{d=5} + \ldots$$

$$\Downarrow$$

$$\mu^2 H^\dagger H$$

The hierarchy problem

$$\mathcal{L}_{SM} = \mathcal{L}^{d=2} + \mathcal{L}(g,\lambda)^{d=4} + \frac{1}{\Lambda}\mathcal{L}^{d=5} + \ldots$$

$$\mu^2 H^\dagger H$$

▲ is it *reasonable* to expect $|\mu^2| \ll \Lambda^2$ **?**

The hierarchy problem

$$\mathcal{L}_{SM} = \mathcal{L}^{d=2} + \mathcal{L}(g,\lambda)^{d=4} + \frac{1}{\Lambda}\mathcal{L}^{d=5} + \ldots$$

$$\mu^2 H^\dagger H$$

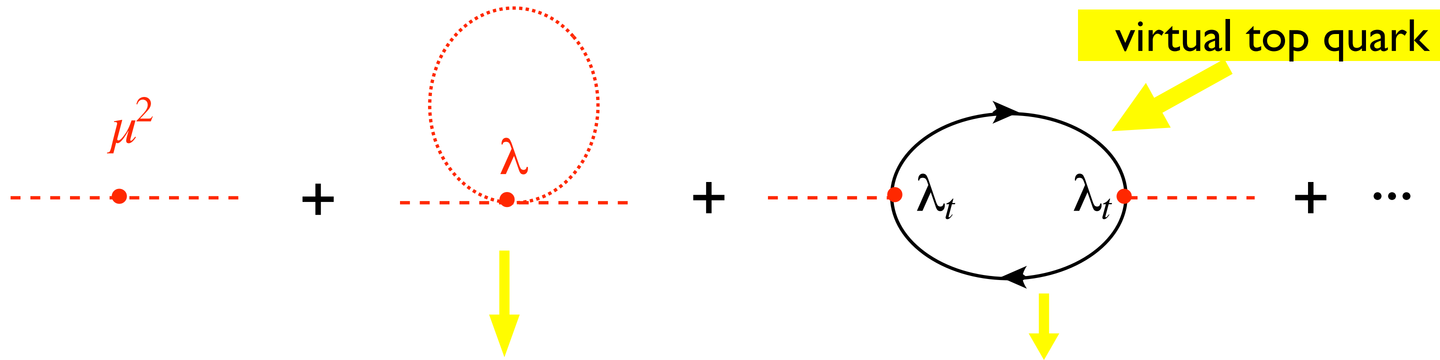
- is it *reasonable* to expect $|\mu^2| \ll \Lambda^2$?

one way to try and answer is to assume a hierarchy exists at tree level:

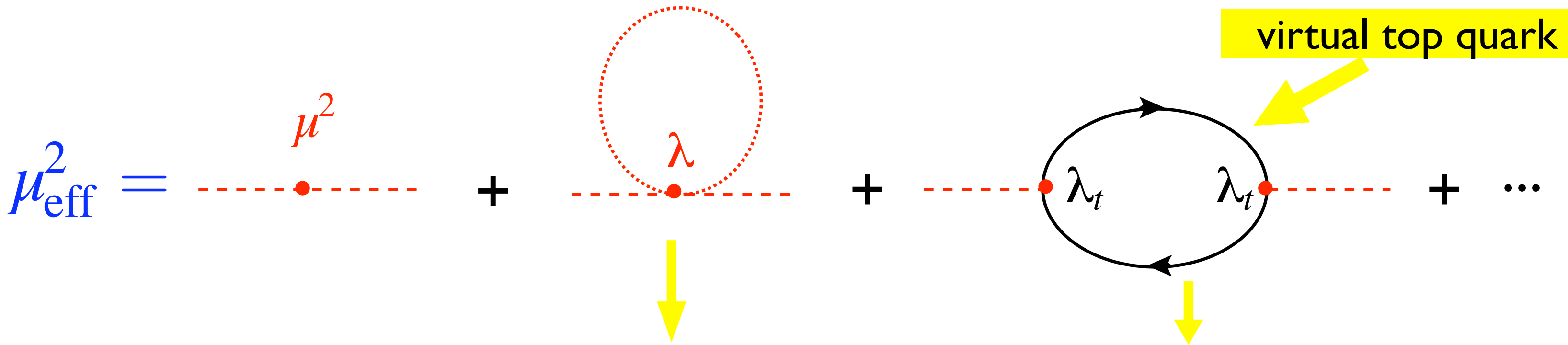
$$|\mu^2_{\rm tree}| \ll \Lambda^2$$

and estimate quantum effects to see if they mantain this hierarchy

$$\mu_{\rm eff}^2 =$$



$$\sim \quad + \frac{3\lambda}{(2\pi)^4}\int \frac{d^4p}{p^2} \qquad - \frac{6\lambda_t^2}{(2\pi)^4}\int \frac{d^4p}{p^2}$$

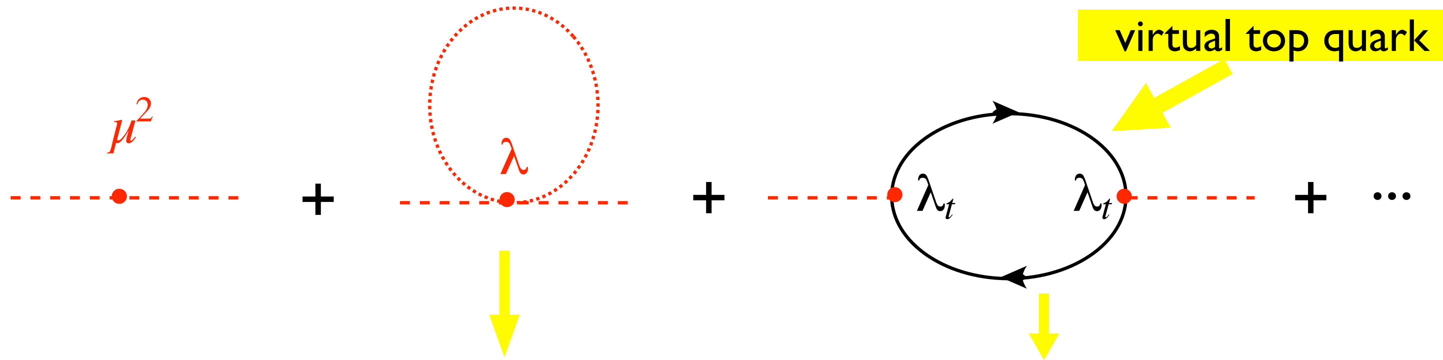


$$\sim \; + \frac{3\lambda}{(2\pi)^4}\int \frac{d^4p}{p^2} \qquad - \frac{6\lambda_t^2}{(2\pi)^4}\int \frac{d^4p}{p^2}$$

$$+ \frac{3\lambda}{16\pi^2}\Lambda^2 \qquad - \frac{3\lambda_t^2}{8\pi^2}\Lambda^2$$

cut-off integral at p ~ Λ

$\mu_{\rm eff}^2 =$



$$\sim \quad + \; \frac{3\lambda}{(2\pi)^4}\int \frac{d^4p}{p^2} \qquad - \; \frac{6\lambda_t^2}{(2\pi)^4}\int \frac{d^4p}{p^2}$$

cut-off integral at p ~ Λ

$$+ \; \frac{3\lambda}{16\pi^2}\Lambda^2 \qquad - \; \frac{3\lambda_t^2}{8\pi^2}\Lambda^2$$

$\mu_{\rm eff}^2$ does not like to stay small when $\Lambda \to \infty$!!

quantum correction to the vacuum energy: top quark contribution

$$\Delta E = -\frac{1}{2}\sum_{i,k}\omega(k) = -\frac{12}{2}\int\sqrt{k^2+m_t^2}\,\frac{d^3k}{(2\pi)^3} =$$

$$= -6\int\left\{k+\frac{m_t^2}{2k}+\cdots\right\}\frac{d^3k}{(2\pi)^3}$$

$$m_t^2 = \lambda_t^2 H^\dagger H$$

$$-H^\dagger H \times \left(\frac{3}{4\pi^2}\lambda_t^2\int dk^2\right)$$

$$\Delta\mu^2$$

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$$m_t^2 = \lambda_t^2 H^\dagger H$$

$$= \quad -\frac{3}{2\pi^2}\int k^2 dk^2 \qquad -H^\dagger H \times \left(\frac{3}{4\pi^2}\lambda_t^2\int dk^2\right)$$

Λ^4 contribution
to vacuum energy !!

$$\Delta\mu^2$$

$$\mu^2_{eff} \quad = \quad \mu^2 + c\Lambda^2$$

large Λ → μ^2 must be tuned to make μ^2_{eff} small

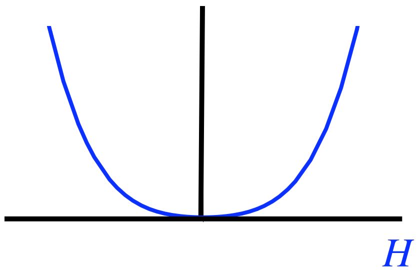
fine-tuning: $$\frac{\mu^2 + c\Lambda^2}{\Lambda^2} \sim \frac{\upsilon_F^2}{\Lambda^2} \overset{\Lambda=10^{15}\mathrm{GeV}}{=} 10^{-30}$$

This is the hierarchy problem

Fermi scale ⟷ Higgs field H vacuum expectation value

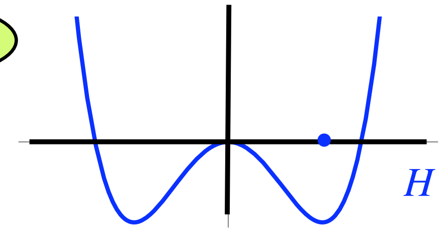
Higgs potential: $$V(H) = m^2 H^2 + \lambda H^4$$

$m^2 > 0$



$< H > = 0$

$m^2 < 0$



$< H > \equiv \upsilon_F = \frac{|m^2|}{2\lambda}$

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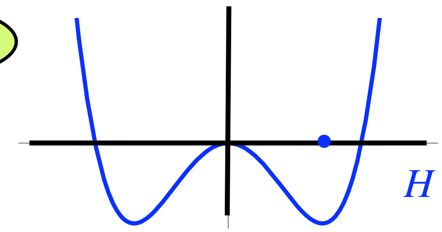

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$m^2 < 0$ in our world:

$$< H > = \upsilon_F$$

gives rise to all other masses

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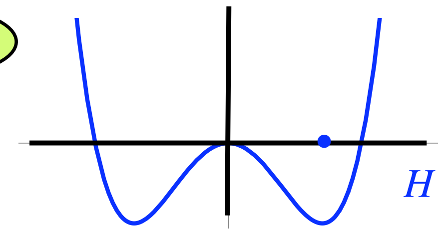

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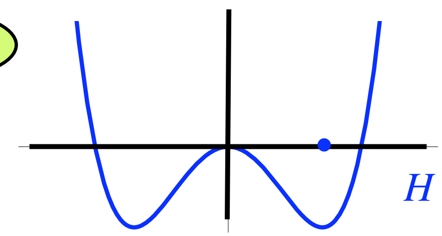

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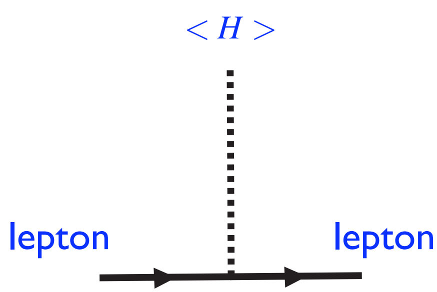
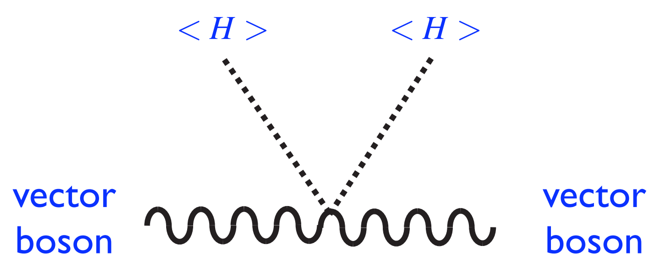


$$m_\ell = \lambda_\ell < H >$$



$$m_W^2 = g^2 < H >^2$$

$$V(H) \quad = \quad m^2 H^2 + \lambda H^4$$

perturbativity $\lambda \lesssim 16\pi^2$ ⇒ $< H > = \sqrt{\frac{-m^2}{2\lambda}} \gtrsim O(\frac{|m|}{4\pi})$

- m^2 picks up all sorts of additive quantum corrections

if SM valid up to Planck scale then it is natural to expect $|m^2| \sim O(M^2_{\text{Planck}})$

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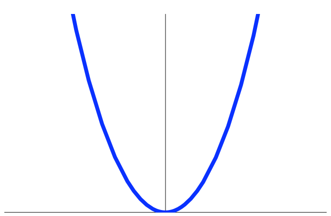
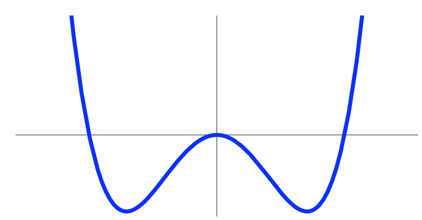
either



$< H > = 0$

or



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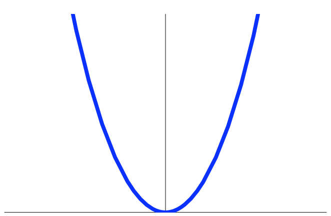
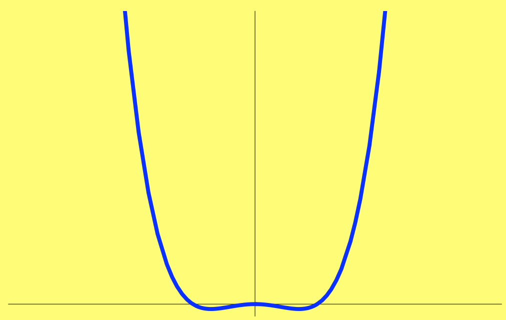


$< H > = 0$

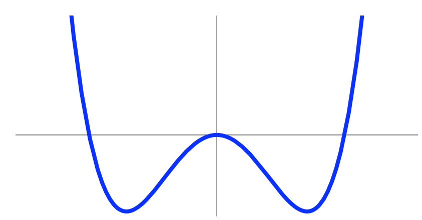
but we need



$< H > = \varepsilon M_{\text{Planck}}$

$\varepsilon \sim 10^{-17}$

or

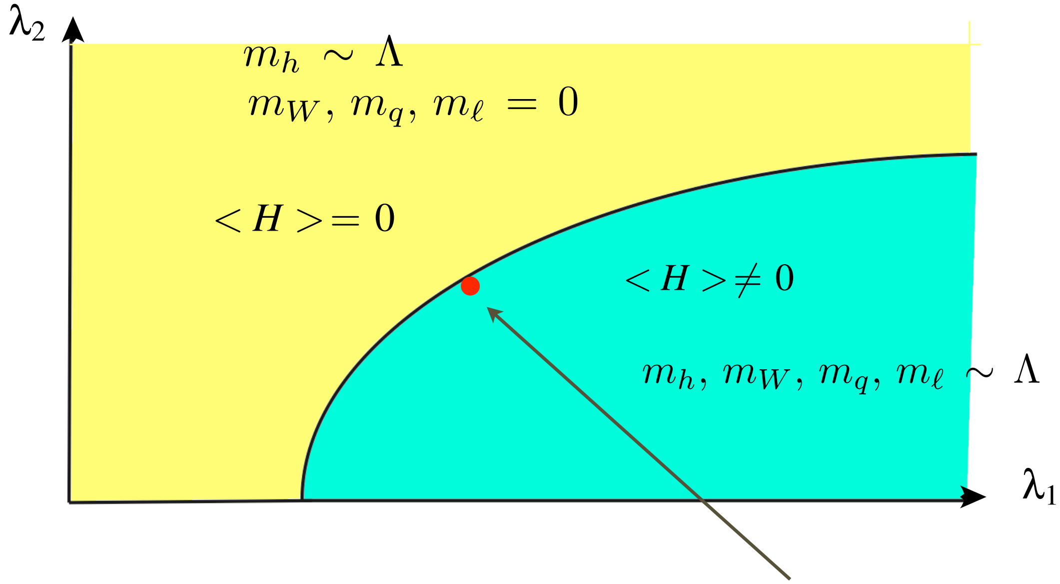


$< H > = O(M_{\text{Planck}})$

Graphical picture of hierarchy puzzle

$$\mathcal{L}_{fund} = \mathcal{L}(g_1, g_2, \Lambda, \ldots; H, W_\mu^I, q, \ell, \ldots)$$

phase diagram



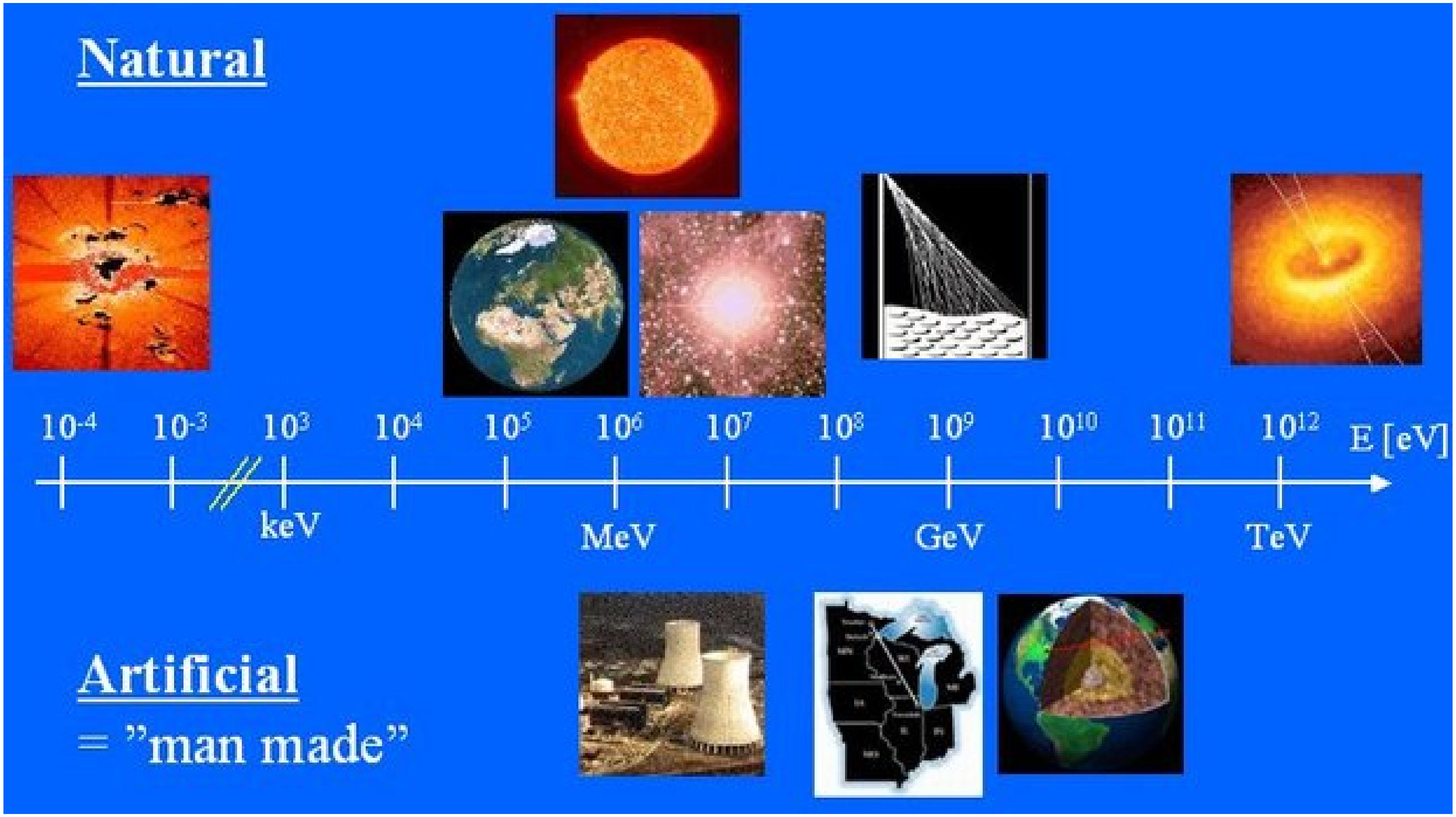
SM lives extremely close to the critical line is

Power divergent effects can be reabsorbed by renormalization of coefficient of lower dimension operators

must exist a scheme where these effects are absent ab initio

Dimensional Regularization

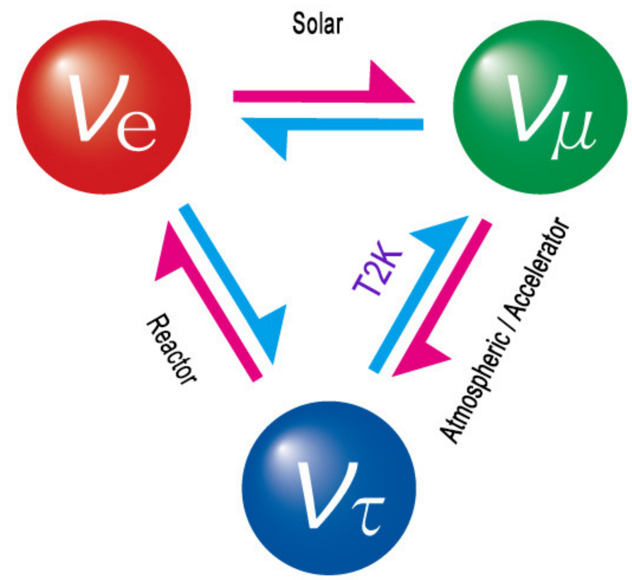
Neutrino physics



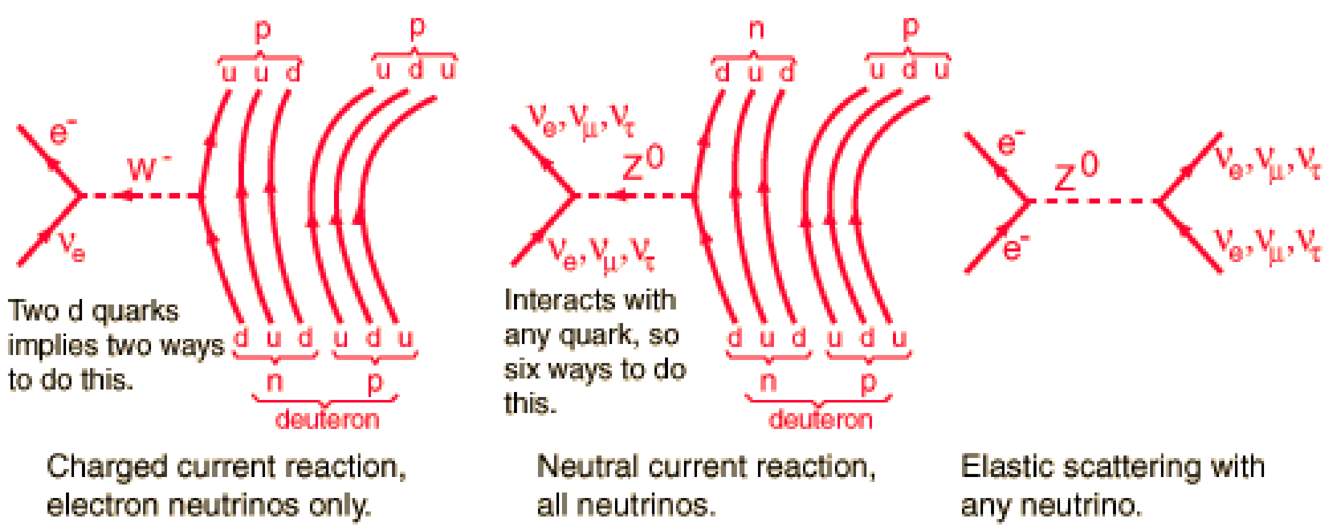
- **Natural:** The Sun; The Earth's atmosphere; Supernovae within our galaxy; The Earth's crust; Cosmic accelerators
- **Man made:** Nuclear power plants; Neutrino superbeams and factories

- **40 years ago:** neutrino were thought **strictly flavour lepton number was conserved** (no $\mu \to e + \gamma$, no $\tau \to eee$, etc.)

- **Today:** neutrino oscillations confirmed by many **independent** experiments (both **appearance** and **disappearance** data)



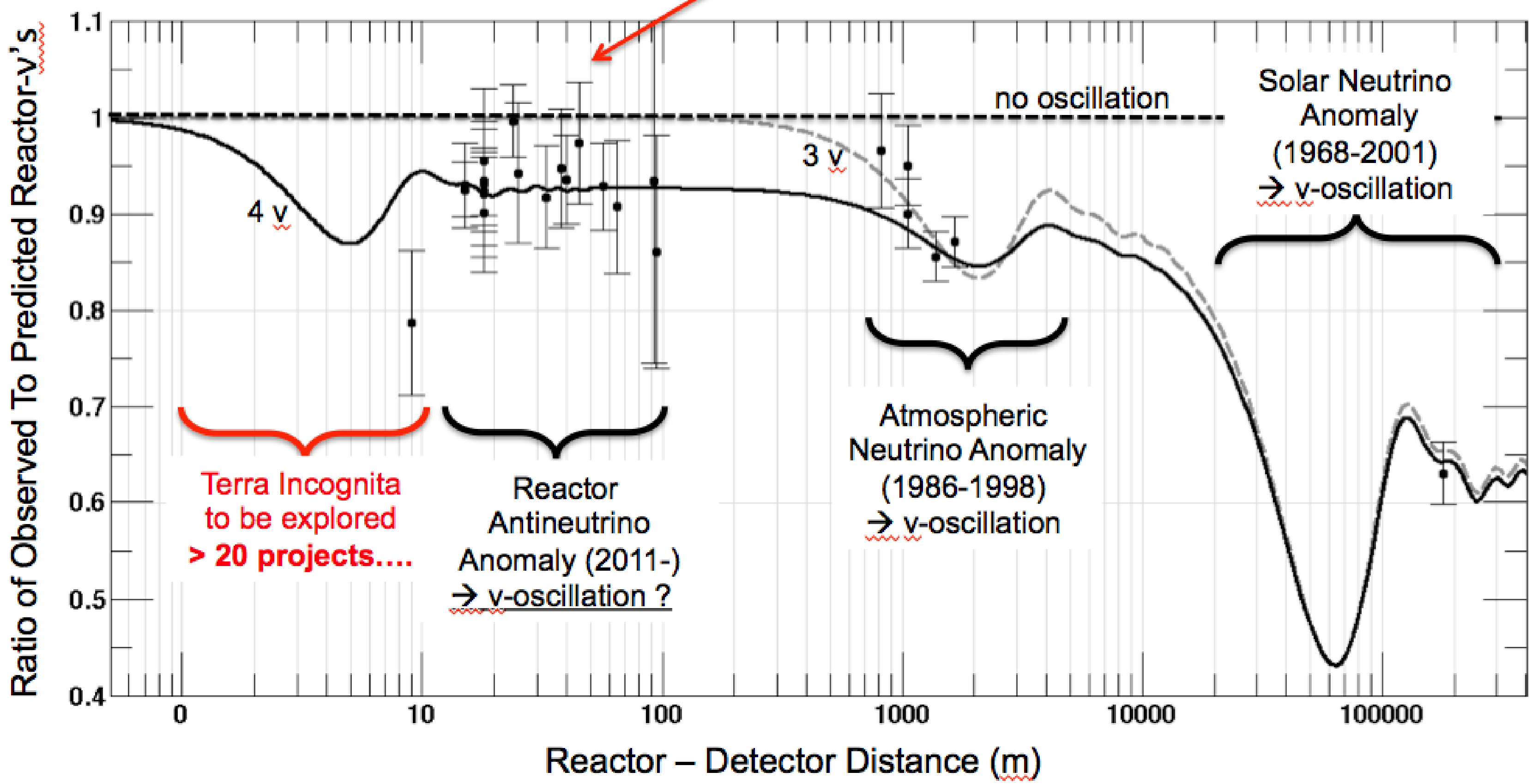
Neutrino oscillation between three generations



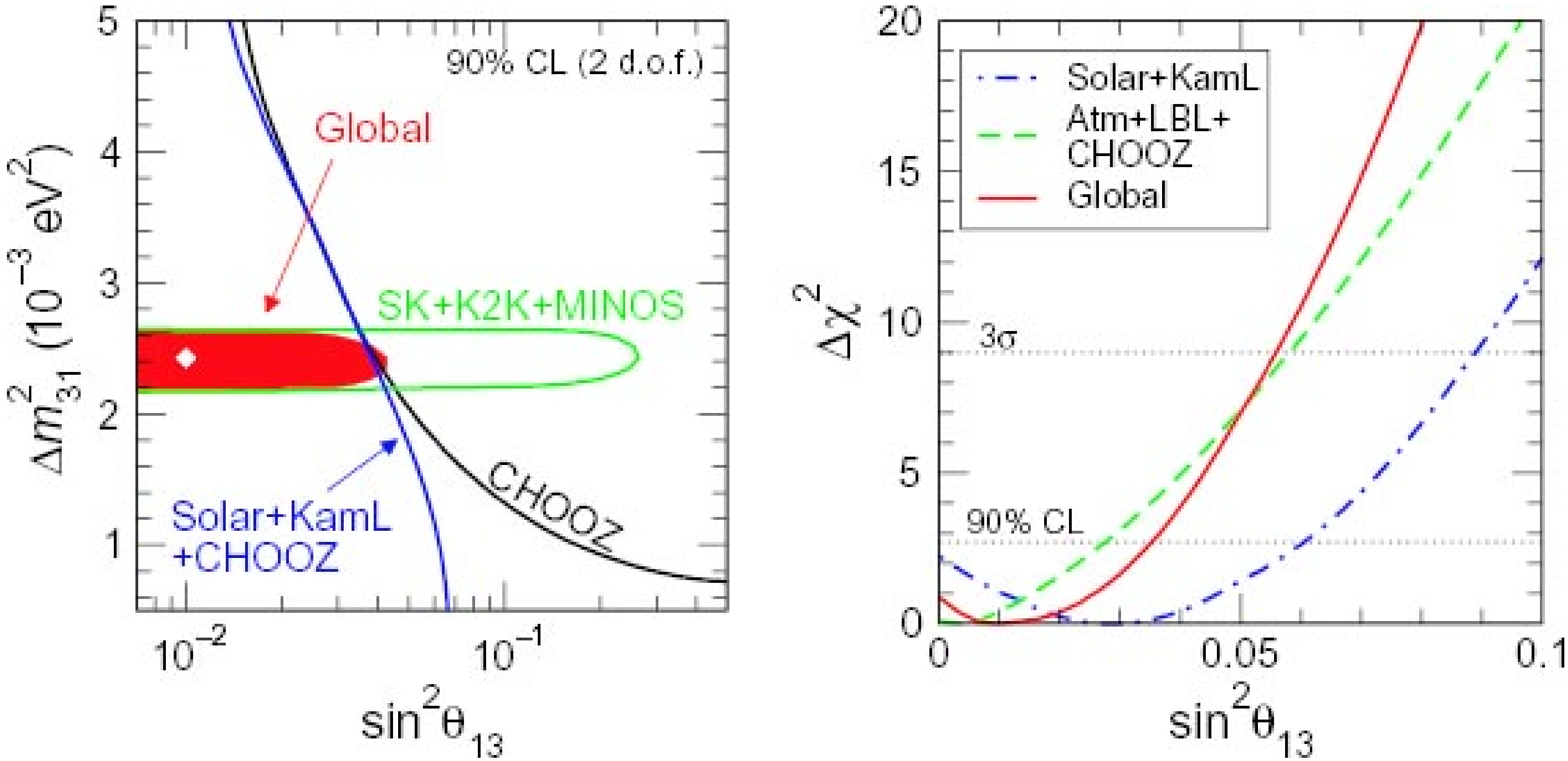
Neutrino detection at SNO

The Reactor Antineutrino Anomaly

- **Observed/predicted averaged event ratio: R=0.927±0.023 (3.0 σ)**

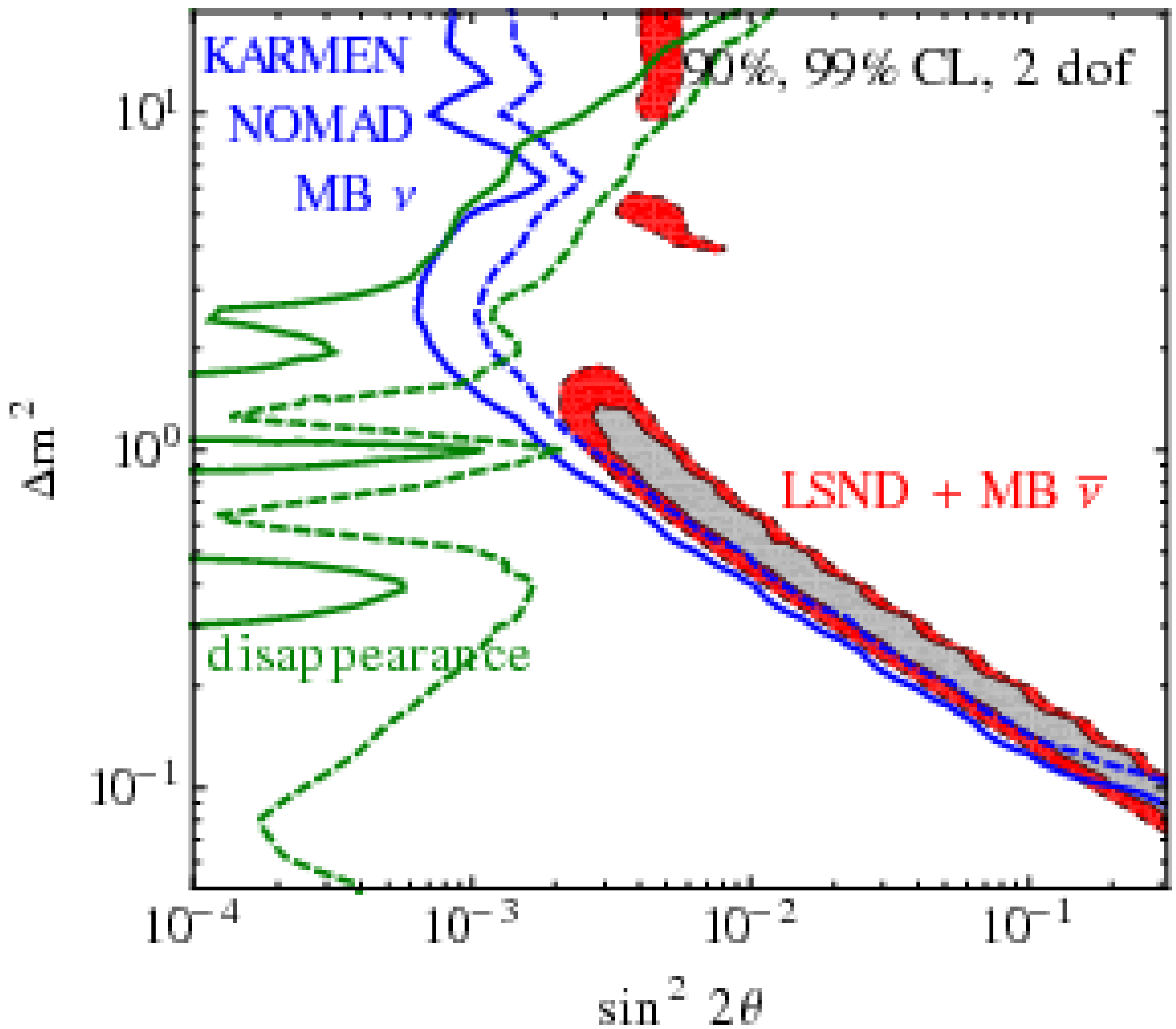


Th. Lasserre – TPC-Paris 2012



- Data from different experiments are consistent and allow to provide a **global fit** to the neutrino oscillations parameters

All anomalies together cannot be true!



All data would be consistent **if** red contours were to the **left** of both green and blue contours

Neutrino oscillations

- Neutrinos are always created or detected with a well defined flavour ν_e, ν_μ, ν_τ $(W^+ \to e^+ + \nu_e)$ – **charge** (or **gauge**, or **flavour**) eigenstates

- Experiments on neutrino oscillations determined two mass differences between neutrino **mass eigenstates**

- This means that there is at least **three** mass states ν_1, ν_2, ν_3

- And there exists a 3×3 unitary transformation U that relates mass eigenstates (ν_1, ν_2, ν_3) to flavour eigenstates $(\nu_e, \nu_\mu, \nu_\tau)$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Neutrino mixing matrix (reminder)

- **Recall** that neutrinos $\nu_{e,\mu,\tau}$ couple to $W^{\pm}$ bosons and to charged leptons (neutrinos are part of SU(2) doublet)

$$\mathcal{L}_{\rm CC} = \bar{\nu}_e \not{W}^+ e^- + \bar{\nu}_\mu \not{W}^+ \mu^- + \dots$$

  Invariant under $\nu_e \to \nu_e e^{i\alpha}$ simultaneously with $e^- \to e^- e^{i\alpha}$, etc.

- All other terms in the Lagrangian have the form $\bar{\psi}\not{D}\psi$ or $m\bar{\psi}\psi$ — i.e. are invariant if $\psi \to \psi e^{i\alpha}$ (here ψ is any of $\nu_e, \nu_\mu, \nu_\tau, e, \mu, \tau$)

- Additionally, we can rotate each of the $\nu_{1,2,3}$ by an independent phase

- 5 of 9 parameters of the mixing matrix U can be absorbed in the redefinitions of $\nu_{1,2,3}$ and $\nu_{e,\mu,\tau}$ (6th phase does is overall redefinition of all fields – does not change U).

Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix

- The rest $9-5=4$ parameters are usually chosen as follows: **3** **mixing angles** $\theta_{12}, \theta_{23}, \theta_{13}$ and **1** **phase** ϕ (since 3×3 real orthogonal matrix has 3 parameters only)

$$U = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12}e^{i\phi} - c_{12}s_{13}s_{23} & c_{12}c_{23}e^{i\phi} - s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{23}s_{12}e^{i\phi} - c_{12}c_{23}s_{13} & -c_{12}s_{23}e^{i\phi} - c_{23}s_{12}s_{13} & c_{13}c_{23} \end{pmatrix} \quad (4)$$

where one denotes $\cos\theta_{12} = c_{12}$, $\sin\theta_{23} = s_{23}$, etc.

Three rotations plus **one** phase ϕ:

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & e^{-i\phi}\sin\theta_{13} \\ 0 & 1 & 0 \\ -e^{i\phi}\sin\theta_{13} & 0 & \cos\theta_{13} \end{pmatrix} \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Dirac and Majorana mass terms

- Mass eigenstates $\nu_{1,2,3}$ are **freely propagating massive fermions**
- Only two types of such fermions are possible which differ by their mass terms:
  - **Dirac mass term** requires **adding new particles** $N_1, N_2, \ldots$:

$$\mathcal{L}_{Dirac} = \begin{pmatrix} \bar{\nu}_1 \\ \bar{\nu}_2 \\ \bar{\nu}_3 \end{pmatrix} \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} + h.c. \quad (5)$$

  - **Majorana mass term:**

$$\mathcal{L}_{\text{Majorana}} = \begin{pmatrix} \nu_1^c \\ \nu_2^c \\ \nu_3^c \end{pmatrix} \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \quad (6)$$

m_1, m_2, m_3 can be complex

- Majorana mass term couples ν and its charge conjugate. Requires no new particles

- However, neutrino is a part of the SU(2) doublet $L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}$ and therefore a Majorana mass term reads in the

$$\bar{\nu}^c_\alpha \nu_\beta \to \frac{c_{\alpha\beta}(\bar{L}_\alpha \cdot \tilde{H}^\dagger)(L_\beta \cdot \tilde{H})}{\Lambda}$$

where Λ is some constant with the dimension of **mass**

- This is an "operator of dimension 5" or "non-renormalizable" interaction

- For many people this was a satisfactory viewpoint: in the logic of **effective field theory** one expects the operator of dimensions 5, 6, etc. whose contributions are **small** at energies $E \ll \Lambda$.

$$\text{Neutrino mass term} = \frac{c_{\alpha\beta}(\bar{L}_\alpha \cdot H^\dagger)(L_\beta \cdot H)}{\Lambda}$$

- Assuming $c_{\alpha\beta} \sim \mathcal{O}(1)$ one gets

$$\Lambda \sim \frac{v^2}{m_{\text{atm}}} \sim 10^{15}\ \text{GeV}$$

- In the logic of EFT one expects that some “heavy” particles had mediated this type of interaction and that at energies $E \lesssim \Lambda$ new particles should appear

../images/Heisenber_Euler-eps-converted-to.pdf

?

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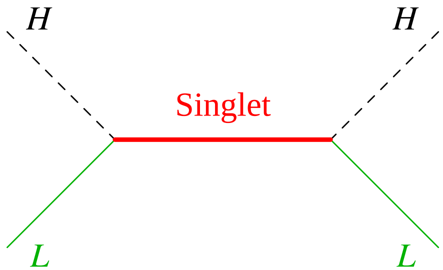
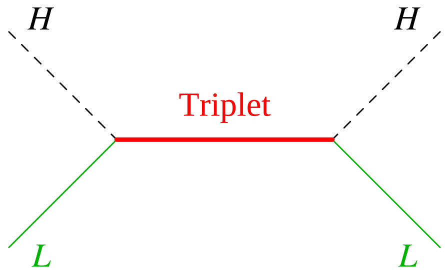
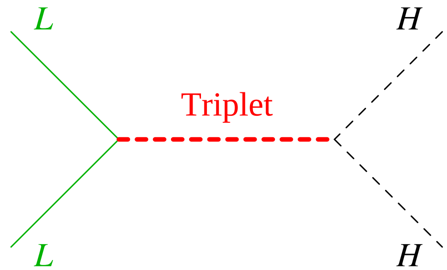
../images/Heisenber_Euler-eps-converted-to.pdf

$$\frac{\alpha^2}{45 m^4}\Big((E^2 - B^2)^2 + 7(E\cdot B)^2\Big)$$

Integrating out massive electrons leads to **Heisenberg-Euler effective action** and light-to-light scattering







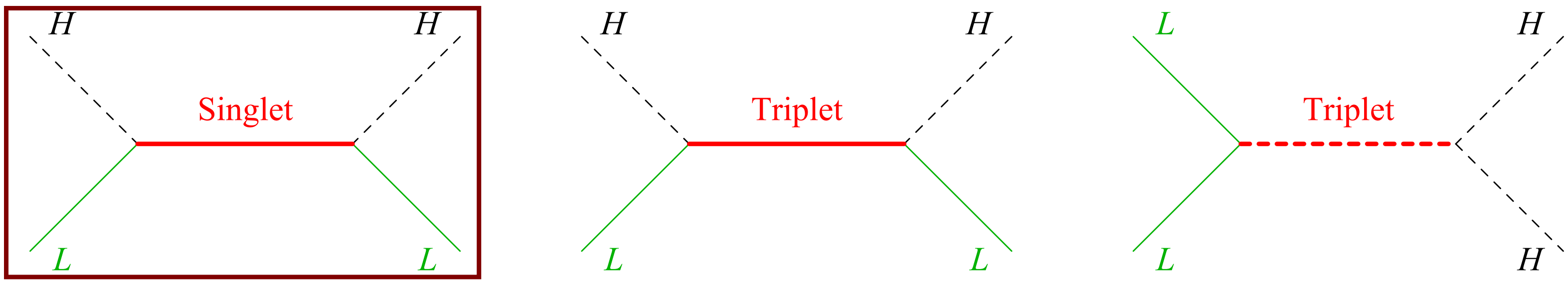
Type I see-saw
extra singlet fermion

Type III see-saw
extra SU(2) triplet fermion
with zero hypercharge

Type II see-saw
extra SU(2) triplet scalar
with hypercharge 1

There are models with “loop mediated neutrino masses”, etc.

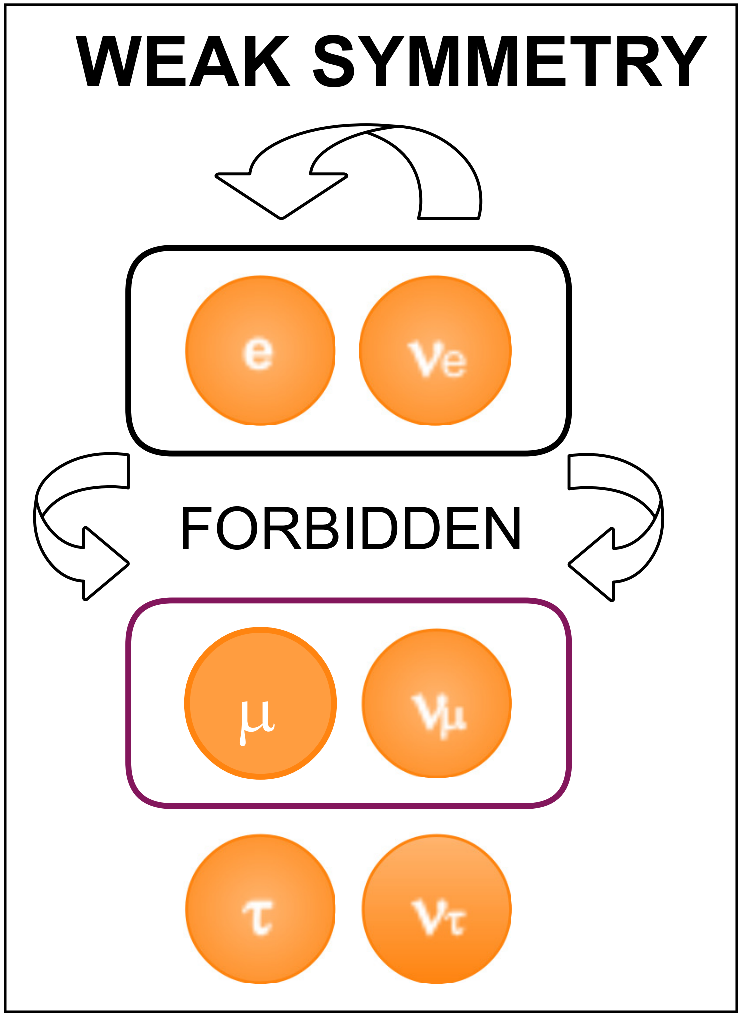
Strumia & Vissani *“Neutrino masses and mixings and. . . ”* [hep-ph/0606054v3]



Type I see-saw	Type III see-saw	Type II see-saw
extra singlet fermion	extra SU(2) triplet fermion with zero hypercharge	extra SU(2) triplet scalar with hypercharge 1

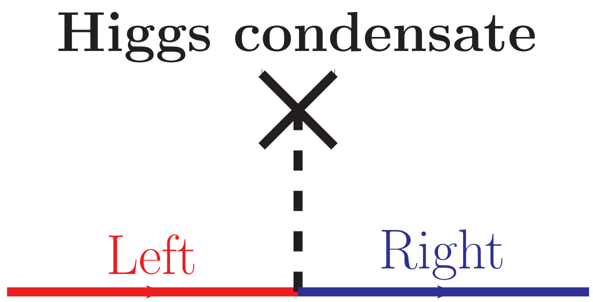
- If neutrino masses are due to **type-I see-saw mechanism**, this implies existence of new particles — sterile neutrinos
- Can they affect any other observables beyond neutrino masses?
- Can they be probed (with "effective energy scale" being 10^{15} GeV)?

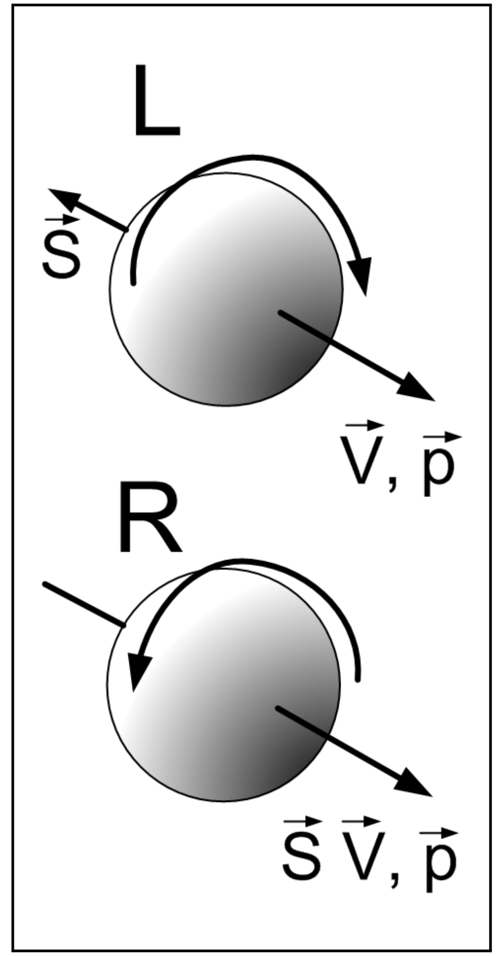
Boyarsky, O.R., Shaposhnikov Ann. Rev. Nucl. Part. Sci. (2009), [0901.0011]



- Number of leptons is conserved in each generation
- i.e. we know with high precision that muons μ cannot convert into electrons e.
- By virtue of the electroweak symmetry neutrinos do not change their types (i.e. $\nu_e \not\leftrightarrow \nu_\mu$)
- To **break symmetry** between electron and neutrino we need **Higgs boson**

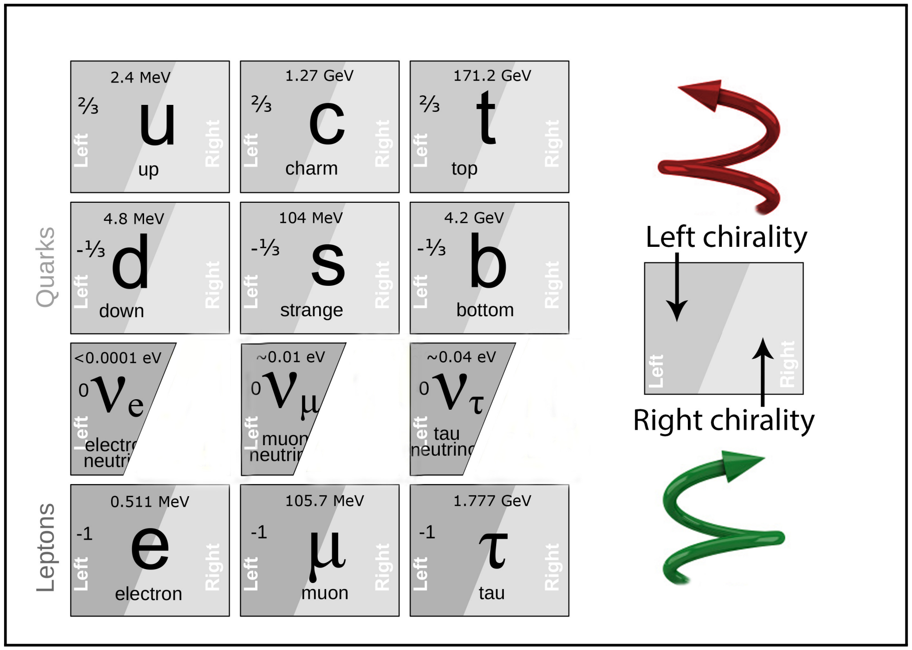
Mass term and Higgs

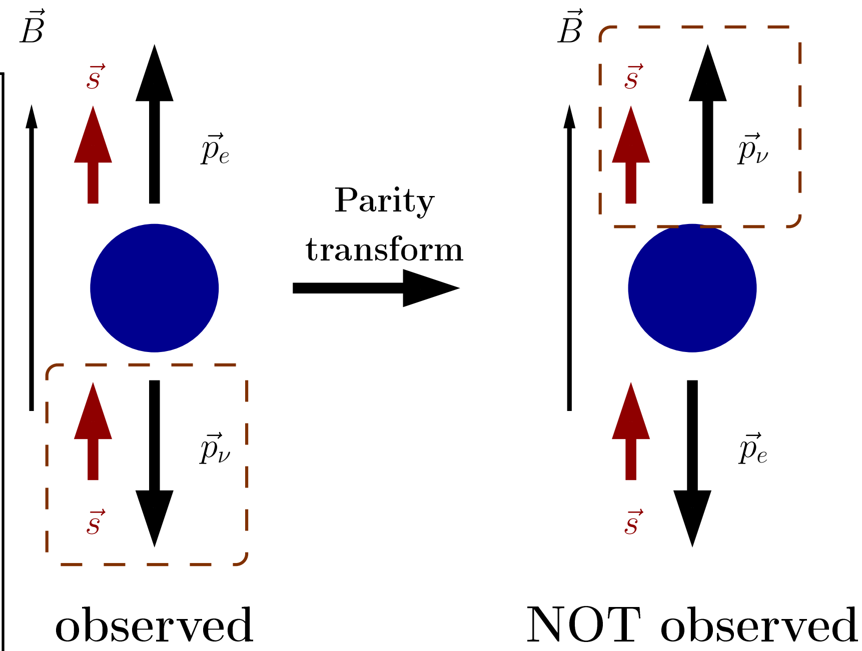




- Higgs boson (spin-0 particle) couples left to right chiralities of fermion
- In the absence of mass term left and right components are independent
- Gauge transformations should rotate left and right by the same phase (otherwise mass term won't be gauge invariant)

$$\bar{\psi}(i\gamma^\mu\partial_\mu - \cancel{m})\psi = \begin{pmatrix}\psi_R^* \\ \psi_L^*\end{pmatrix}\begin{pmatrix} \cancelto{0}{-m} & i(\partial_t + \vec{\sigma}\cdot\vec{\nabla}) \\ i(\partial_t - \vec{\sigma}\cdot\vec{\nabla}) & \cancelto{0}{-m}\end{pmatrix}\begin{pmatrix}\psi_L \\ \psi_R\end{pmatrix} = 0$$

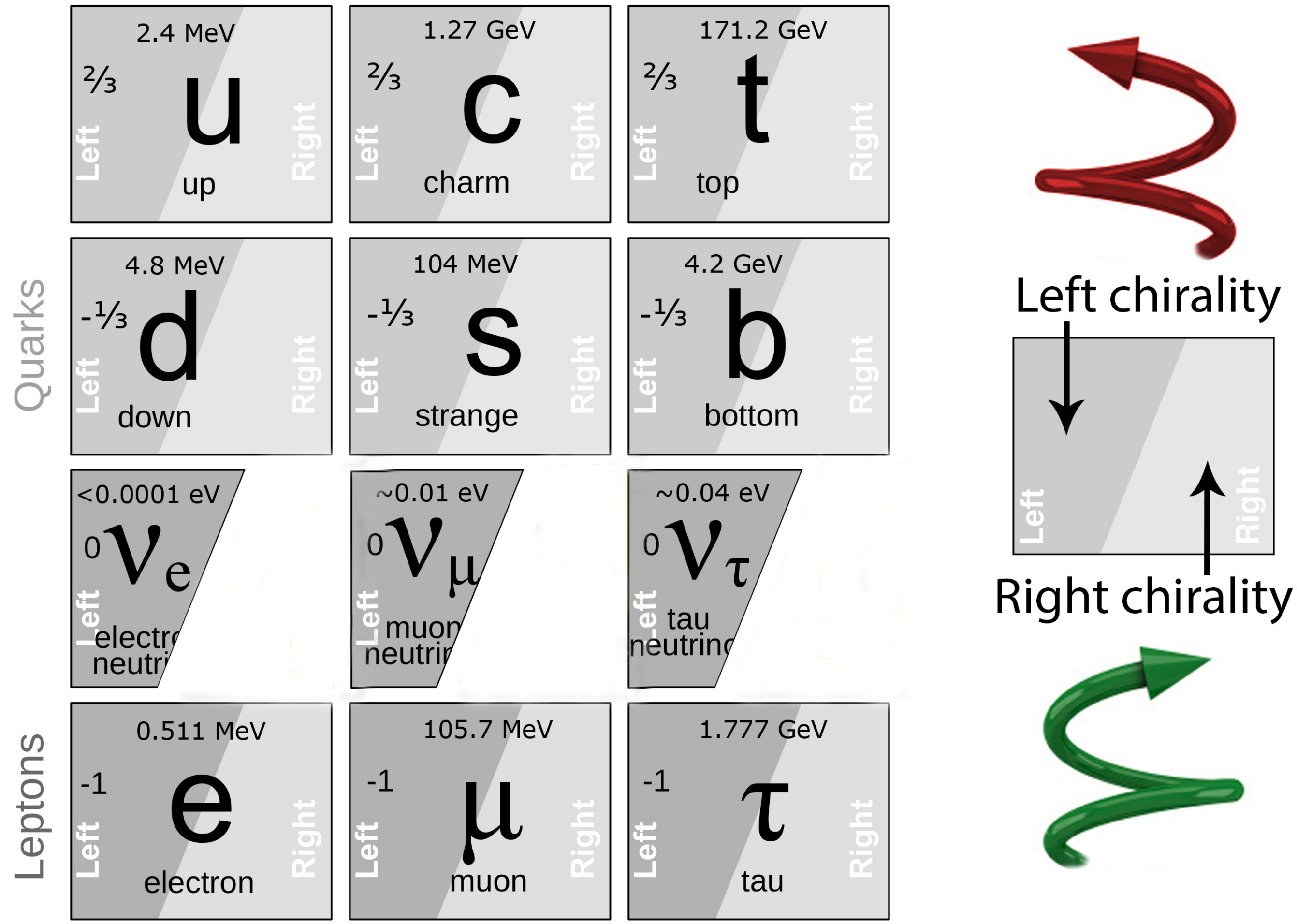




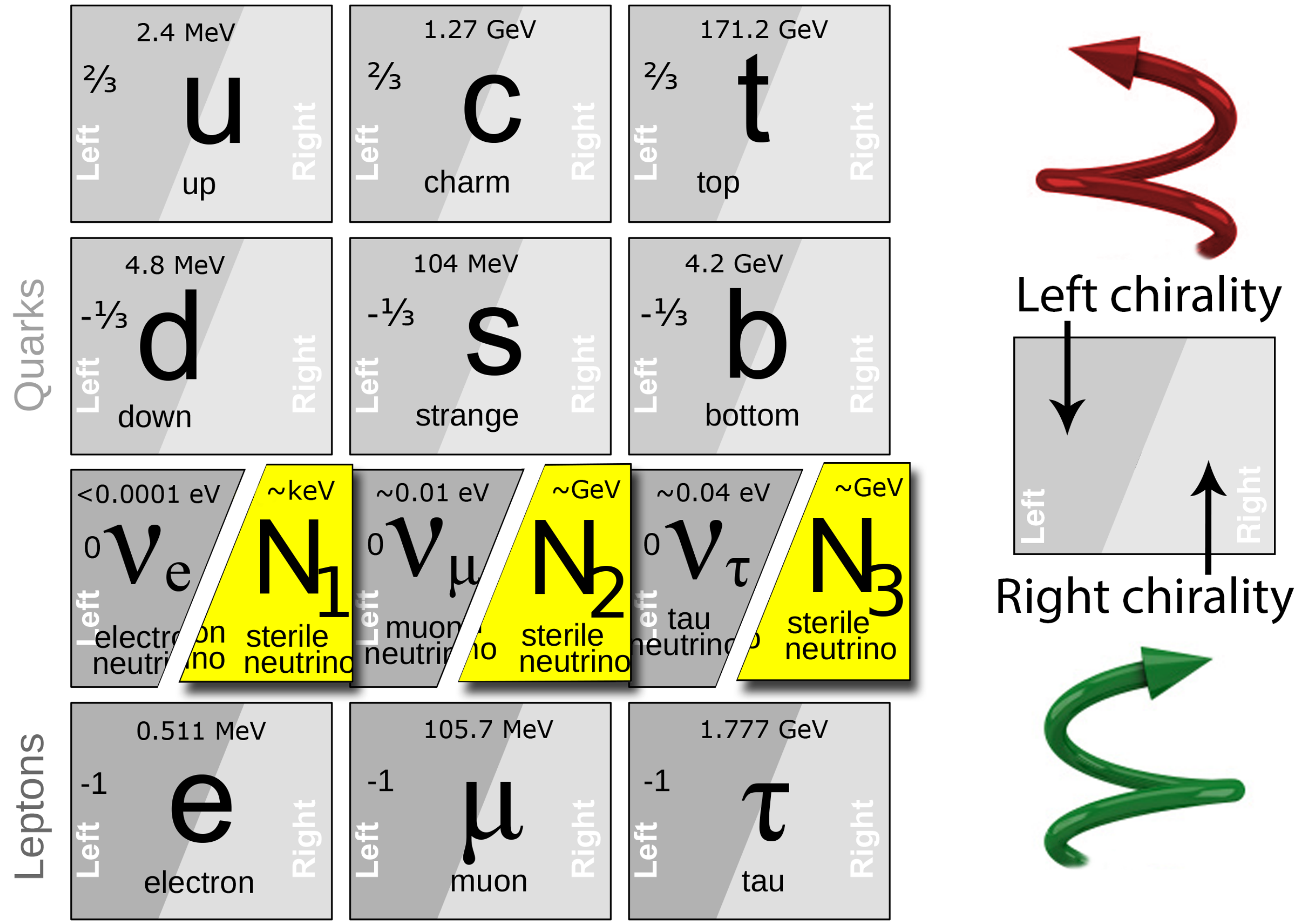
http://pearl1.lanl.gov/external/atom_trap/parity.htm

Unlike **all other fermions in the Standard Model**, neutrinos are **always left-polarized**

Oscillations ⇒ new particles!



Oscillations ⇒ new particles!



Right components of neutrinos?!

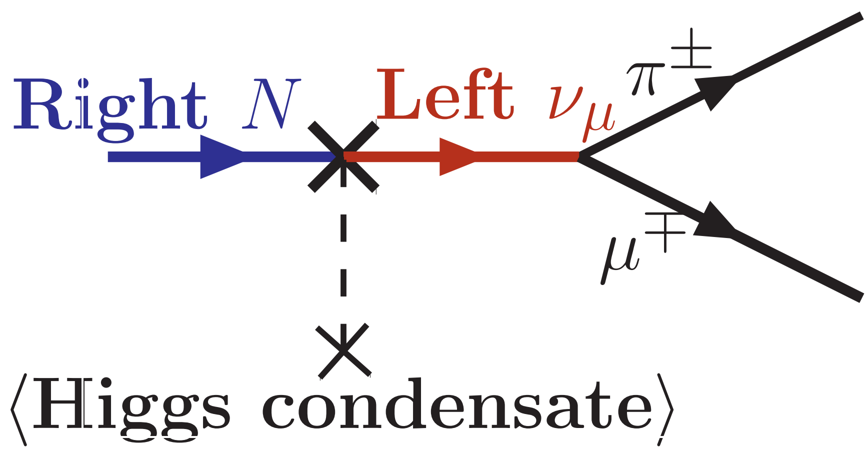
- New particles (N_1, N_2, N_3) carry no charges with respect to known interactions

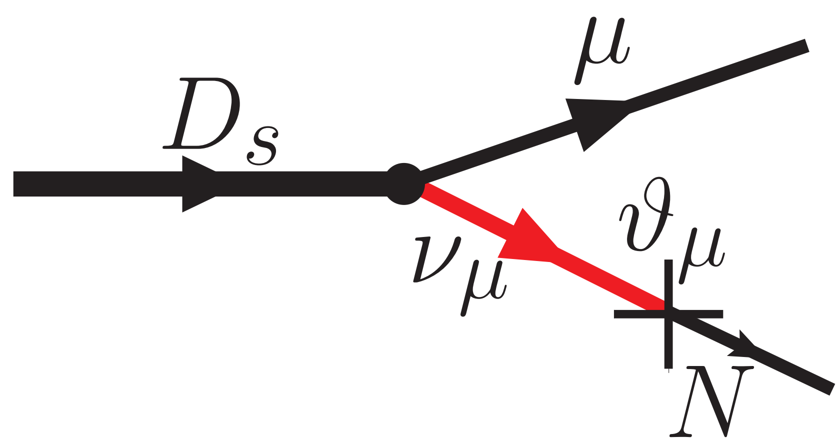
that is why they are often called **sterile neutrinos**

- They have different mass from left neutrinos

that is why they are sometimes called **heavy neutral leptons**

- They are **heavier** than ordinary neutrinos but interact **much weaker**





Sterile neutrinos behave as **superweakly interacting** massive neutrinos with a smaller Fermi constant $\vartheta \times G_F$

- This **mixing strength** or **mixing angle** is

$$\vartheta^2_{e,\mu,\tau} \equiv \frac{|M_{\text{Dirac}}|^2}{M^2_{\text{Majorana}}} = \frac{\mathcal{M}_{\text{active}}}{M_{\text{sterile}}} \approx 5 \times 10^{-11} \left(\frac{1\ \text{GeV}}{M_{\text{sterile}}}\right)$$


WANTED
Massive neutral particle
Interacting weaker than neutrino

Sterile neutrinos may provide all conditions necessary for successful baryogenesis:

- Their "weaker-than-weak" interaction (ϑG_F) means that they go out of equilibrium much earlier than even neutrinos
- Their mass matrix may contain additional CP-violating phases (*a la* CP violating phases of CKM matrix)
- Their Majorana masses violate lepton number

This class of scenarios is called LEPTOGENESIS

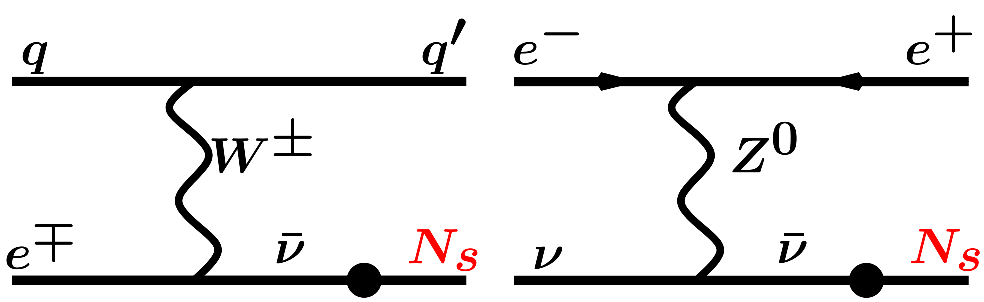
- **Sterile neutrino** is a new neutral particle, interacting **weaker-than-neutrino**

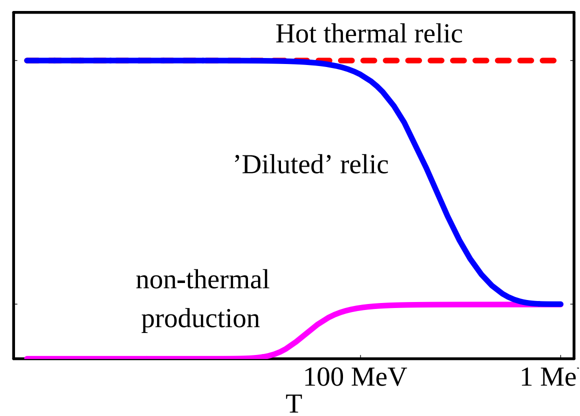
- Never was in thermal equilibrium in the early Universe $\Rightarrow$

Dodelson & Widrow'93; Dolgov & Hansen'00

$\Rightarrow$ Its abundance **slowly builds up** but **never reaches the equilibrium** value

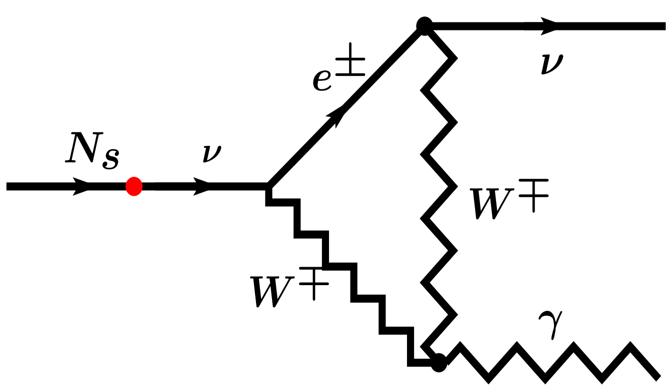
$\Rightarrow$ avoids Tremaine-Gunn-like bound





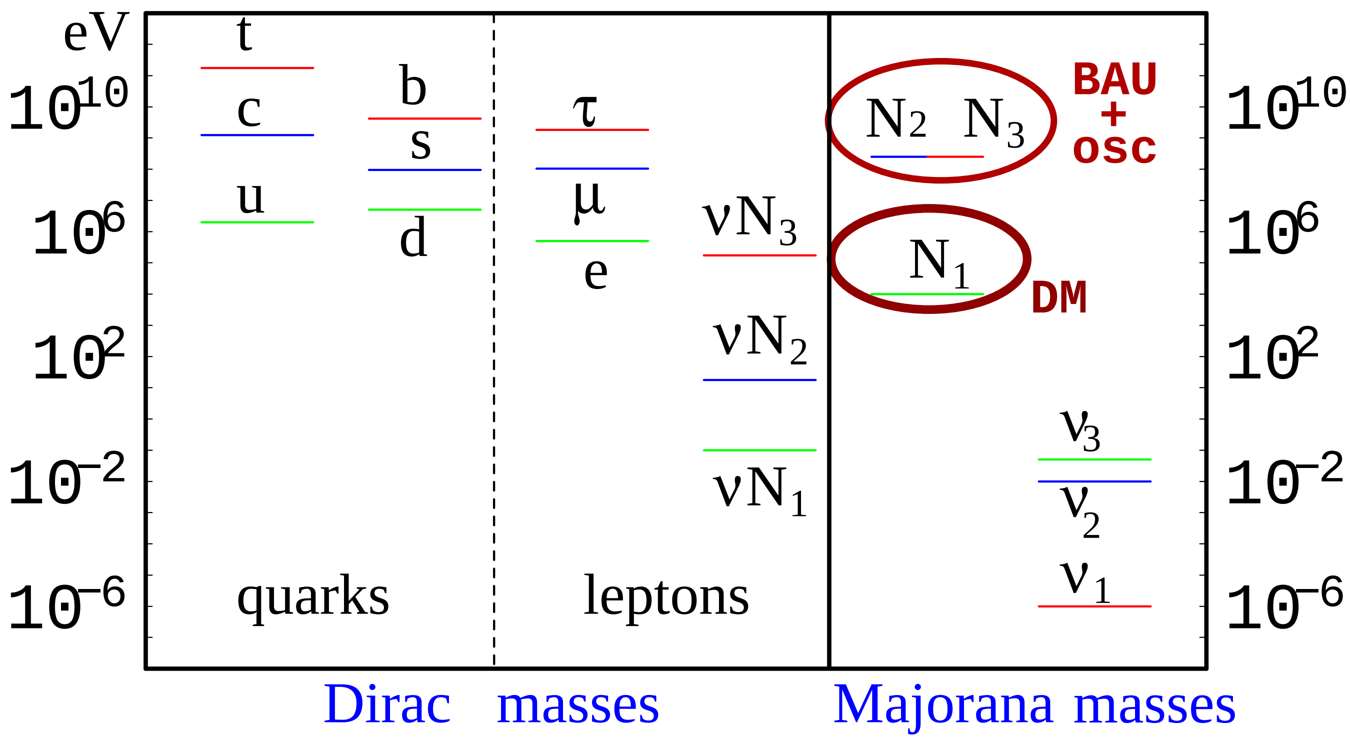
- Once every $\sim 10^8 \,\text{div}\, 10^{10}$ scatterings a sterile neutrino is created instead of the active one

- Very hard/impossible to search at LHC
- Very hard/impossible to search in laboratory experiments
- Can be **decaying** with the lifetime exceeding the age of the Universe
- Can we detect such a rare decay?
- **Yes!** if you multiply the probability of decay by a large number – amount of DM particles in a galaxy (typical amount $\sim 10^{70}$–10^{100} particles)



One assumption about physics behind neutrino oscillations (existence of new particles N_1, N_2, N_3) may also explain the existence of dark matter and matter-antimatter asymmetry of the Universe

Particles of the νMSM



Masses of sterile neutrinos as those of other leptons
Yukawas as those of electron or smaller

Neutrino Minimal Standard Model == νMSM

νMSM predicts that this picture holds up to the "Planck scale energies" (when Compton wavelength $\sim$ Schwarzschild radius of particle)

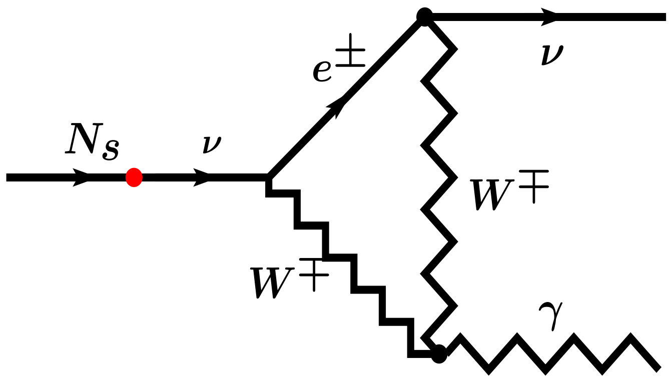
How to test this model?

Decaying dark matter signal

- Can be **decaying** with the lifetime exceeding the age of the Universe

- Can we detect such a rare decay?

- **Yes!** if you multiply the probability of decay by a large number – amount of DM particles in a galaxy (typical amount $\sim 10^{70}$–10^{100} particles)



- Two-body decay into two massless particles (DM $\to \gamma + \gamma$ or DM $\to \gamma + \nu$) $\Rightarrow$ narrow decay line

$$E_\gamma = \frac{1}{2} m_{\text{DM}} c^2$$

- The width of the decay line is determined by **Doppler broadening**