Discrete symmetries of nature

Discrete Symmetries

- Space, time translation & orientation symmetries are all *continuous* symmetries
  - Each symmetry operation associated with one or more continuous parameter
- There are also *discrete* symmetries
  - Spatial sign flip (x,y,z → -x,-y,-z) : P
  - Charge sign flip (Q → -Q) : C
  - Time sign flip (t → -t) : T
- Are these discrete symmetries exact symmetries that are observed in nature?
  - Is the assignment of the label (anti) particle a convention or not?
  - Is there a fundamental difference between left-handed and right-handed?

Quantity		*P*	*C*	*T*
Space vector	$\mathbf{x}$	$-\mathbf{x}$	$\mathbf{x}$	$\mathbf{x}$
Time	t	t	t	$-t$
Momentum	$\mathbf{p}$	$-\mathbf{p}$	$\mathbf{p}$	$-\mathbf{p}$
Spin	$\mathbf{s}$	$\mathbf{s}$	$\mathbf{s}$	$-\mathbf{s}$
Electrical field	$\mathbf{E}$	$-\mathbf{E}$	$-\mathbf{E}$	$\mathbf{E}$
Magnetic field	$\mathbf{B}$	$\mathbf{B}$	$-\mathbf{B}$	$-\mathbf{B}$

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In particle physics:

$$P\left|e_L^-\right\rangle = \left|e_R^-\right\rangle$$
$$P\left|\pi^0\right\rangle = -\left|\pi^0\right\rangle$$
$$P\left|n\right\rangle = +\left|n\right\rangle$$

$$C\left|e_L^-\right\rangle = \left|e_L^+\right\rangle$$
$$C\left|u\right\rangle = \left|\bar{u}\right\rangle$$
$$C\left|d\right\rangle = \left|\bar{d}\right\rangle$$
$$C\left|\pi^0\right\rangle = +\left|\pi^0\right\rangle$$

note: the definition of a 'left handed' particle will follow in 'a few slides' time

- For a spinor

$$\psi = u(p)e^{-ipx}$$

the following spinor

$$\psi_c = C\psi^* = (Cu)e^{+ip\cdot x} \tag{1}$$

is also a solution of the Dirac equation:

$$(\not{\partial} - m)\psi_c = e^{+ip\cdot x}(-p_\mu\gamma^\mu - m)Cu^* \stackrel{?}{=} 0 \tag{2}$$

iff the matrix C is chosen in such a way that

$$-\gamma^\mu C = C(\gamma^\mu)^* \tag{3}$$

(and also $C^2 = 1$)

- One possible solution: choose all gamma-matrices **imaginary**. Then $C = \mathbb{1}$ and charge conjugation $=$ complex conjugation

- The corresponding matrices have the form:

$$\gamma^0 = \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} i\sigma^3 & 0 \\ 0 & i\sigma^3 \end{pmatrix}$$

$$\gamma^2 = \begin{pmatrix} 0 & -\sigma^2 \\ \sigma^2 & 0 \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} -i\sigma^1 & 0 \\ 0 & -i\sigma^1 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix}.$$

- Define
$$\gamma_5 \equiv i\gamma_0\gamma_1\gamma_2\gamma_3$$

- Using $\gamma_0^2 = \mathbb{1}$ and $\gamma_i^2 = -\mathbb{1}$ as well as anti-commuting properties of γ-matrices, we find that $(i\gamma_0\gamma_1\gamma_2\gamma_3)(i\gamma_0\gamma_1\gamma_2\gamma_3) = \mathbb{1}$

- Notice that from (3) we find that $\gamma_\mu^* = -C\gamma_\mu C$ and as a result

$$\begin{aligned}\gamma_5^* =& (-i)\gamma_0^*\gamma_1^*\gamma_2^*\gamma_3^* \\ =& (-i)(-C\gamma_0 C)(-C\gamma_1 C)(-C\gamma_2 C)(-C\gamma_3 C) \\ =& -C\gamma_5 C\end{aligned} \tag{4}$$

- Consequence: γ_5 is imaginary in the **Majorana basis** (where $C = \mathbb{1}$)

- Consequence: if γ_5 is real (e.g. in the **Weyl basis**) then

$$\{\gamma_5, C\} = 0 \tag{5}$$

P- and C-conjugations together

- How do C and P operate combined?
- Let ψ_R be right-chiral fermion, i.e.

$$\gamma_5 \psi_R = \psi_R \tag{6}$$

- Charge conjugated fermion $(\psi_R)^c = (C\psi_R^*)$
- What is the parity of charge-conjugated fermion?

$$\gamma_5 (C\psi_R^*) \overset{\text{Eq. (4)}}{=\!=\!=} -C(\gamma_5^* \psi_R^*) \overset{\text{Eq. (6)}}{=\!=\!=} -C(\gamma_5 \psi_R)^* = -C\psi_R^* = -(\psi_R)^c$$

- Let's understand this in different basises:
  - Weyl basis: γ_5 is real matrix and this is a consequence of Eq. (5)
  - Majorana basis: γ_5 is imaginary and this is just a conseqeuence of the complex conjugation

- Recall that the operator of spin $\vec{\Sigma}$ is given by:

$$\Sigma_i = \frac{1}{8}\epsilon_{ijk}[\gamma^j, \gamma^k] = \frac{1}{4}\epsilon_{ijk}\gamma^j\gamma^k \tag{7}$$

demonstrate, that Σ_i form $so(3)$ algebra

- In particular $\Sigma_z = \frac{1}{2}\gamma_1\gamma_2$
- Let $\Sigma_z\psi = \frac{1}{2}\psi$ and let $\psi^c = (C\psi^*)$ be charge conjugated spinor
- One sees that charge conjugation leaves spin **unchanged**

$$\begin{aligned}\Sigma_z\psi^c =&\frac{1}{2}\gamma_1\gamma_2 C\psi^* = \frac{1}{2}C(-C\gamma_1 C)(-C\gamma_2 C)\psi^* = C(\frac{1}{2}\gamma_1\gamma_2\psi)^* \\ =&C(\Sigma_z\psi)^* = \frac{1}{2}C\psi^* = \frac{1}{2}\psi^c\end{aligned} \tag{8}$$

Helicity of particles

- Charge conjugation leaves spin and momentum changed. Therefore **helicity** (i.e. the sign of projection of spin onto momentum) is not affected by C:
  - Left-helical particle $\xrightarrow{C}$ Left-helical antiparticle
  - Right-helical particle $\xrightarrow{C}$ Right-helical antiparticle

CPT theorem

"Any Lorentz-invariant local quantum field theory is invariant under the combined application of *C*, *P* and *T* "

G. Lüders, W. Pauli (1954); J. Schwinger (1951)

Assumptions:

1. Lorentz invariance
2. "principle of locality"
3. Causality
4. Vacuum lowest energy
5. Flat space-time
6. Point-like particles

Consequences:

1. Relation between spin and statistics: fields with integer spin commute and fields with half-numbered spin anticommute; Pauli exclusion principle
2. Particles and antiparticles have **equal mass and lifetime**, equal magnetic moments with opposite sign, and **opposite quantum numbers**

Is parity conserved in nature?

The θ-τ puzzle

Observation of something(s) which decay to two pions and three pions, but whatever decays (now known as K^+), has, in both decays, the same lifetime, mass, spin=0...

In 1953, Dalitz argued that since the pion has parity of -1,

- two pions (*) would combine to produce a net parity of (-1)(-1) = +1,
- and three pions (*) would combine to have total parity of (-1)(-1)(-1) = -1.

Hence, if conservation of parity holds, there are two *distinct* particles with parity +1 (the 'θ') and parity -1 (the 'τ')(**).

But how to explain the fact that the mass and lifetime are the same?

$\boxed{K^{\pm}}$ $\qquad I(J^P) = \frac{1}{2}(0^-)$

K^+ DECAY MODES

K^- modes are charge conjugates of the modes below.

	Mode	Fraction (Γ_i/Γ)	Scale factor/ Confidence level
	Hadronic modes		
Γ_9	$\pi^+\pi^0$	(21.13 ± 0.14) %	S=1.1
Γ_{10}	$\pi^+\pi^0\pi^0$	(1.73 ± 0.04) %	S=1.2
Γ_{11}	$\pi^+\pi^+\pi^-$	(5.576 ± 0.031) %	S=1.1

Citation: S. Eidelman *et al.* (Particle Data Group), Phys. Lett. B **592**, 1 (2004) (URL: http://pdg.lbl.gov)

(*) produced in the decay of a spin=0 mother

PHYSICAL REVIEW VOLUME 104, NUMBER 1 OCTOBER 1, 1956

Question of Parity Conservation in Weak Interactions*

T. D. LEE, *Columbia University, New York, New York*

AND

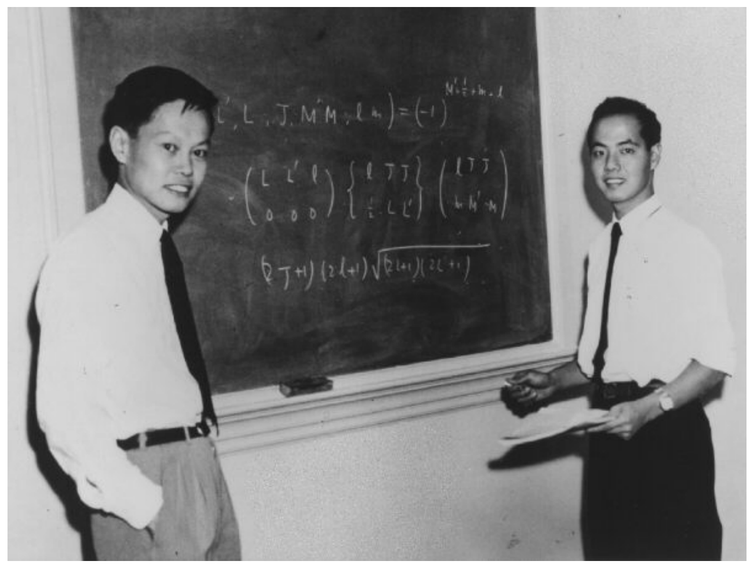
C. N. YANG,† *Brookhaven National Laboratory, Upton, New York*

(Received June 22, 1956)

The question of parity conservation in β decays and in hyperon and meson decays is examined. Possible experiments are suggested which might test parity conservation in these interactions.

RECENT experimental data indicate closely identical masses[1] and lifetimes[2] of the $\theta^+(\equiv K_{\pi 2}{}^+)$ and the $\tau^+(\equiv K_{\pi 3}{}^+)$ mesons. On the other hand, analyses[3] of the decay products of τ^+ strongly suggest on the grounds of angular momentum and parity conservation that the τ^+ and θ^+ are not the same particle. This poses a rather puzzling situation that has been extensively discussed.[4]

One way out of the difficulty is to assume that parity is not strictly conserved, so that θ^+ and τ^+ are two different decay modes of the same particle, which necessarily has a single mass value and a single lifetime. We wish to analyze this possibility in the present paper against the background of the existing experimental evidence of parity conservation. It will become clear that existing experiments do indicate parity conservation in strong and electromagnetic interactions to a high degree of accuracy, but that for the weak interactions (i.e., decay interactions for the mesons and hyperons, and various Fermi interactions) parity conservation is so far only an extrapolated hypothesis unsupported by experimental evidence. (One might even say that the present $\theta-\tau$ puzzle may be taken as an indication that parity conservation is violated in weak interactions. This argument is, however, not to be taken seriously because of the paucity of our present knowledge concerning the nature of the strange particles. It supplies rather an incentive for an examination



The Nobel Prize in Physics 1957

"for their penetrating investigation of the so-called parity laws which has led to important discoveries regarding the elementary particles"

The Exprimental (Re)Solution...

Experimental Test of Parity Conservation in Beta Decay*

C. S. Wu, *Columbia University, New York, New York*

AND

E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson, *National Bureau of Standards, Washington, D. C.*

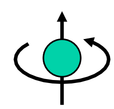
(Received January 15, 1957)

Idea for experiment in collaboration with Lee and Yang: *Look at spin of decay products of polarized radioactive nucleus*

- Production mechanism involves exclusively weak interaction

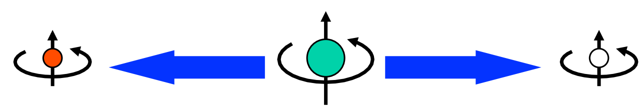
Mme. Chien-Shiung Wu

Parity & Spin



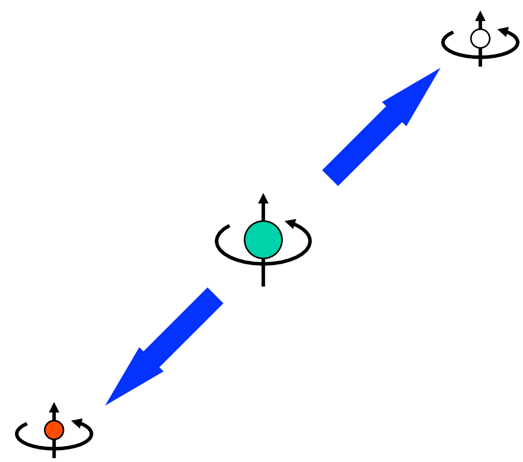
Parity & Spin

- A possible orientation



Parity & Spin

- A possible orientation
- And another...



Parity & Spin

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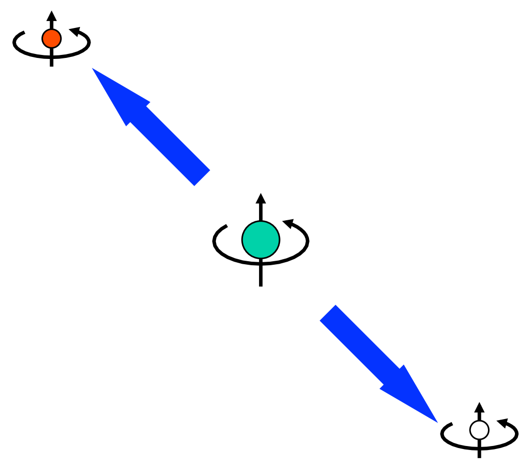
- And another...
- And another...



Parity & Spin

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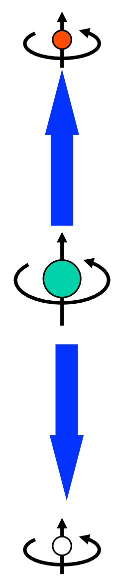
- And another...
- And another...



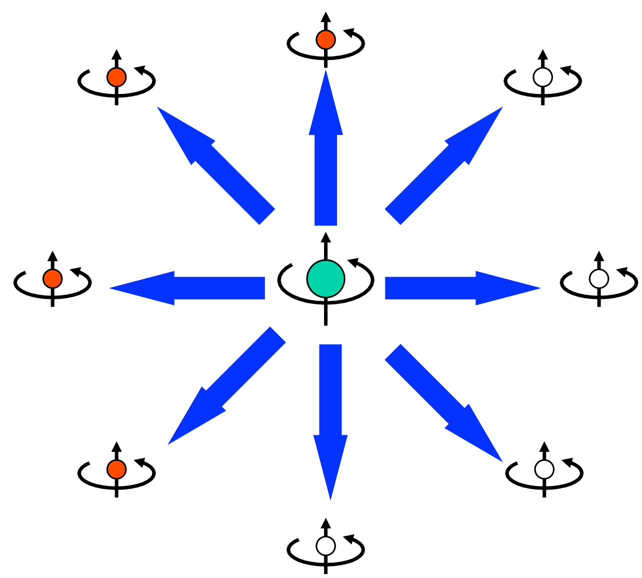
Parity & Spin: Helicity

- A possible orientation
- And another...
- And another...
- Introduce projection of spin on momentum, the helicity, to distinguish:

$$H = \frac{\vec{S}\cdot\vec{P}}{\left|\vec{S}\cdot\vec{P}\right|}$$

- Under parity transform H→-H
- If parity conserved, no reason to favour one value of H over another

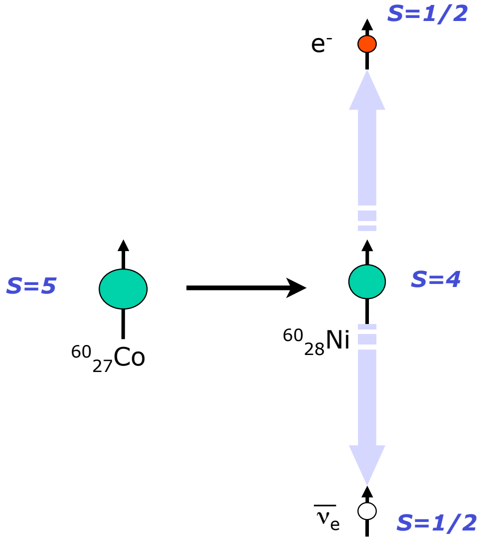
$H = +1$ "Right Handed"

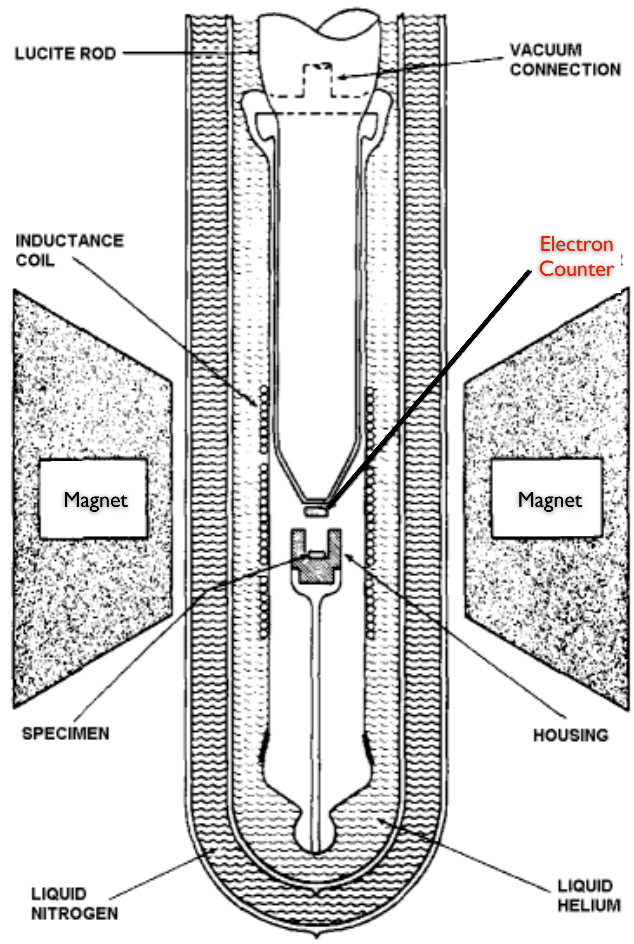


$H = -1$ "Left Handed"

warning:
helicity assignment is not Lorentz invariant for massive particles: an observer can boost 'past' such that p changes direction.

Mme Wu's Experiment : setup

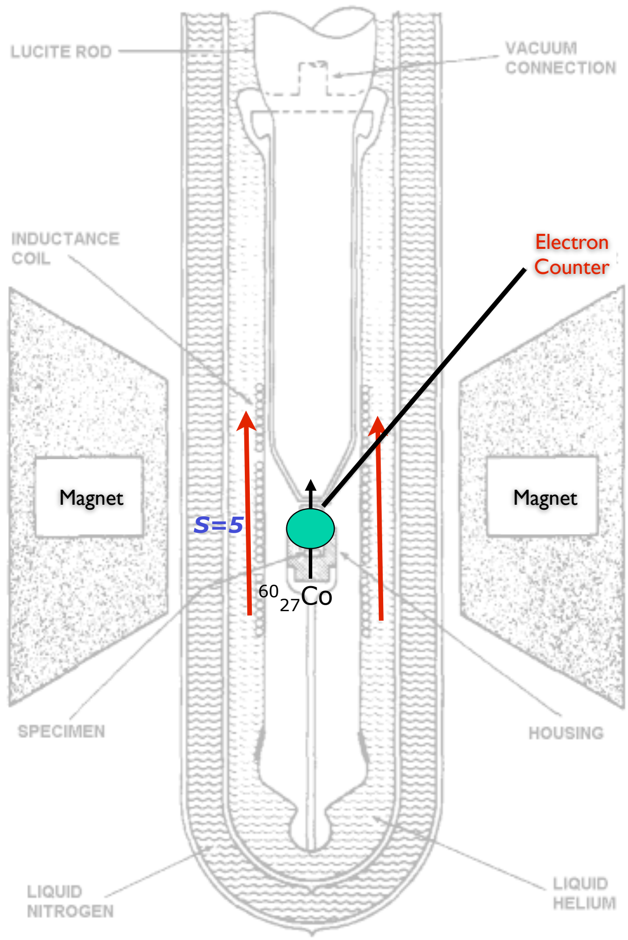




- How do you obtain a sample of ^{60}Co with spins aligned in one direction, and compare to non-aligned case?

- Adiabatic demagnitization of ^{60}Co in a magnetic field at very low temperatures (~0.01 K!). Extremely challenging in 1956!

Mme Wu's Experiment : setup



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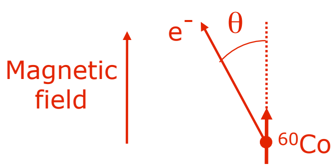

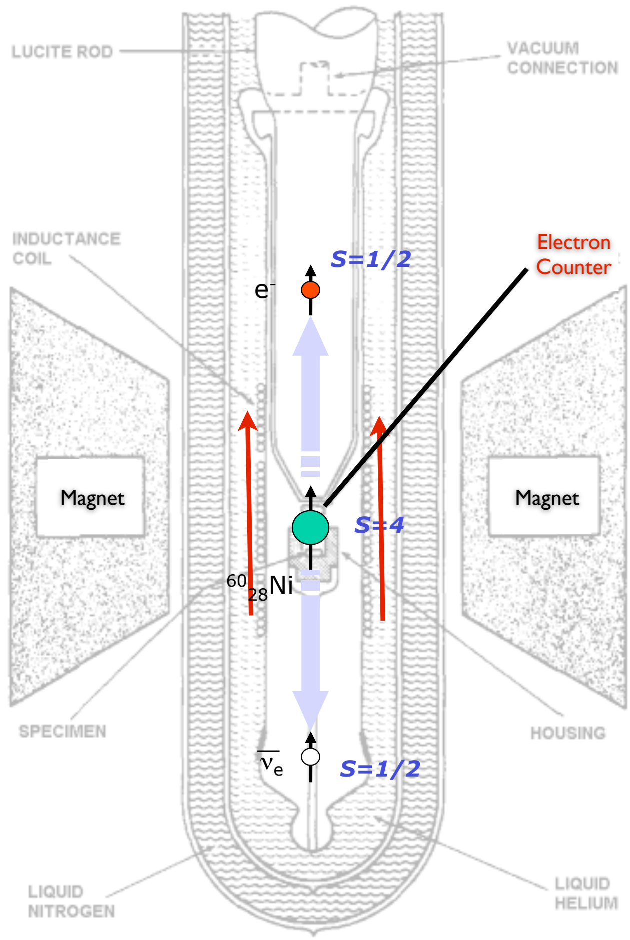

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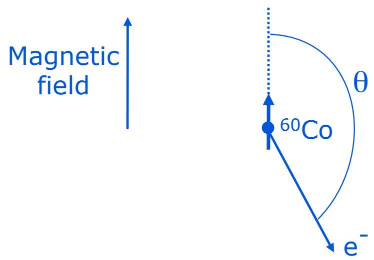

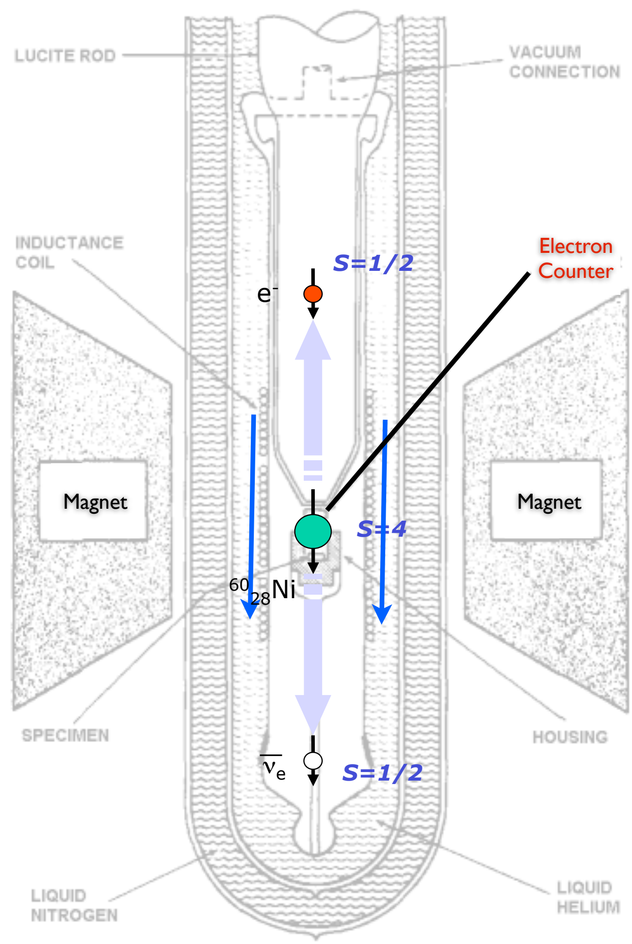

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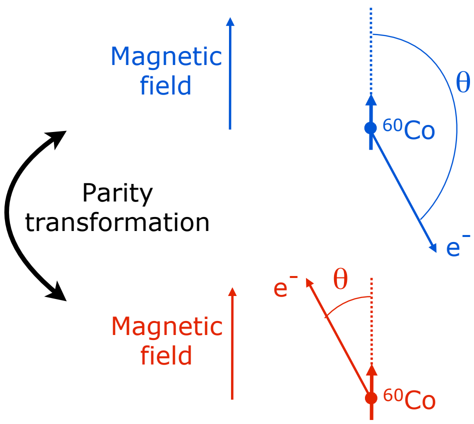
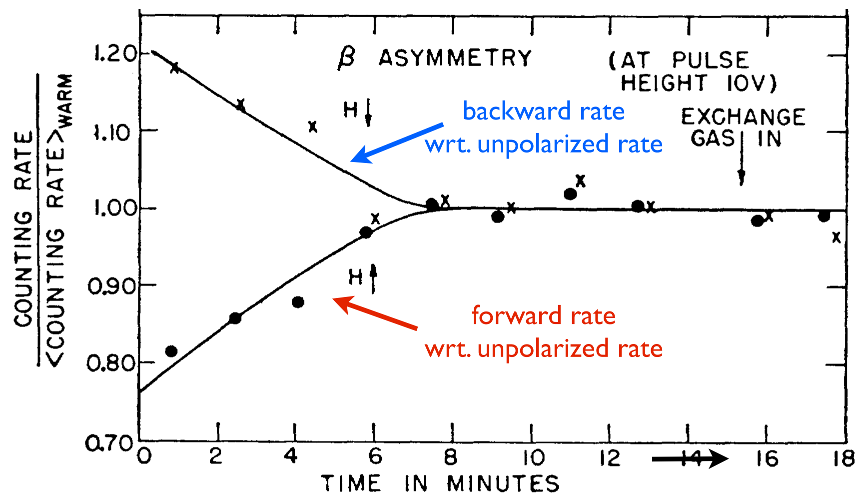
Mme Wu's Experiment : result

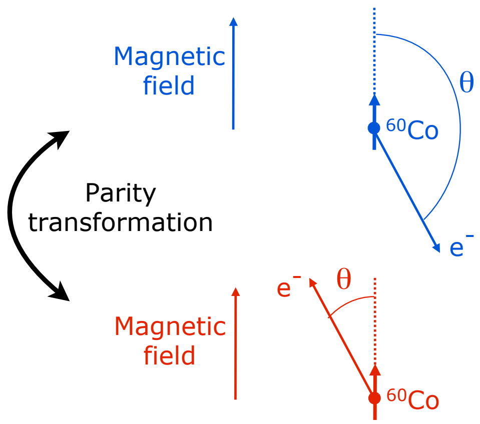


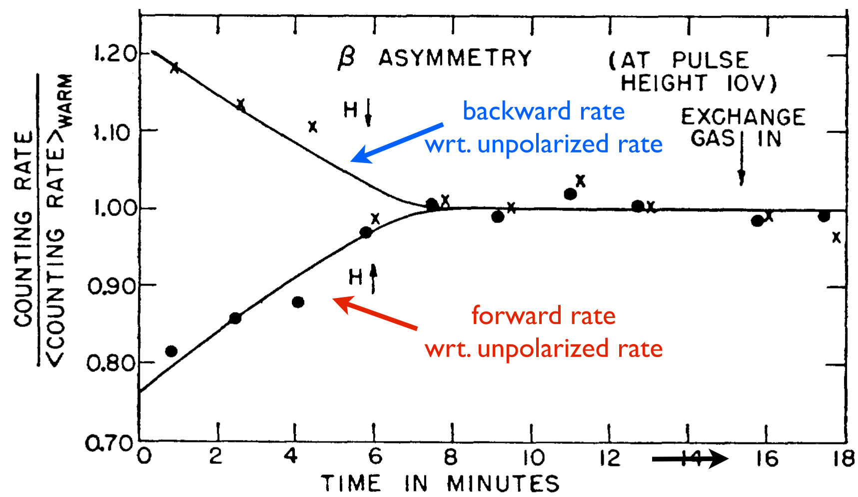


^{60}Co polarization decreases as a function of time as the temperature increases

- The counting rate in the polarized case is different from the unpolarized case
- Changing the direction of the B-field changes the counting rate!
- Electrons are preferentially emitted in the direction opposite the ^{60}Co spin!

Mme Wu's Experiment : conclusion





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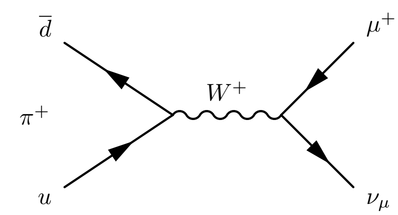
- The counting rate in the polarized case is different from the unpolarized case
- Changing the direction of the B-field changes the counting rate!
- Electrons are preferentially emitted in the direction opposite the ^{60}Co spin!
- Analysis of the results shows that data consistent with the emission of only left-handed (i.e. H = -1) electrons ....
- ... and thus only *right-handed anti-neutrinos*

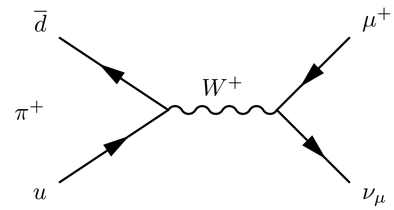
Charge conjugation and CP

From P to C,P and CP



- Lederman et al.: Look at decay $\pi^+ \rightarrow \mu^+ \nu_\mu$

From P to C,P and CP



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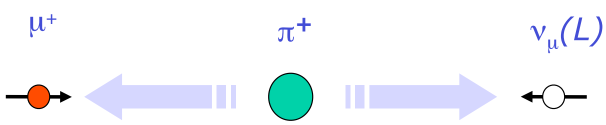
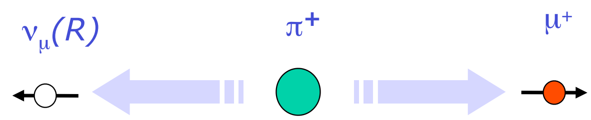
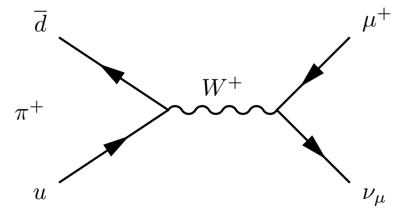




- Pion has spin 0; μ,ν_μ both have spin ½
  → spin of decay products must be *oppositely* aligned
  → Helicity of muon is the *same* as that of neutrino.

From P to C,P and CP



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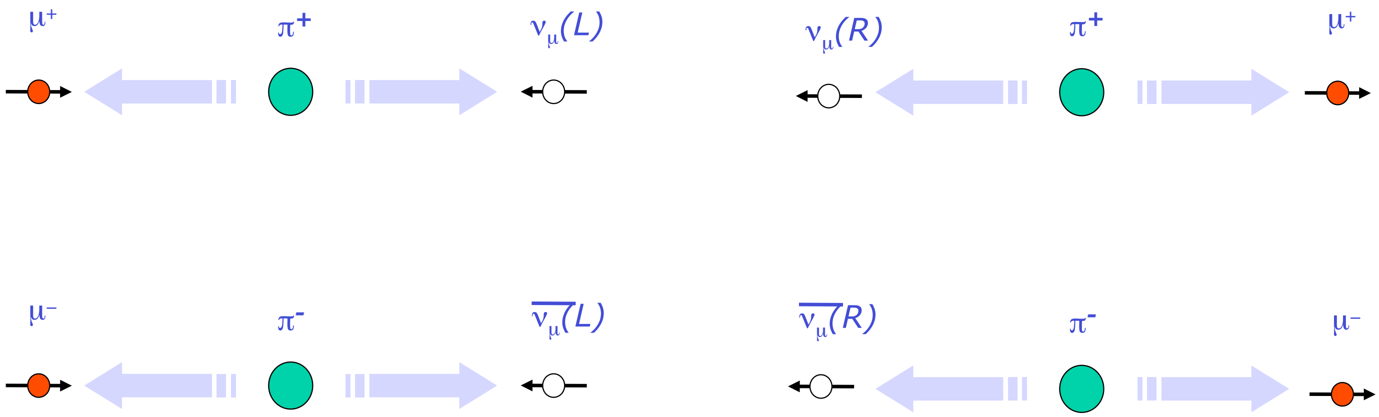

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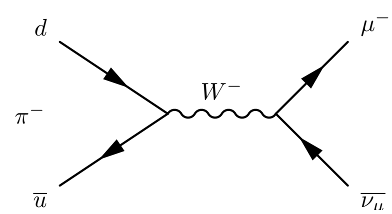
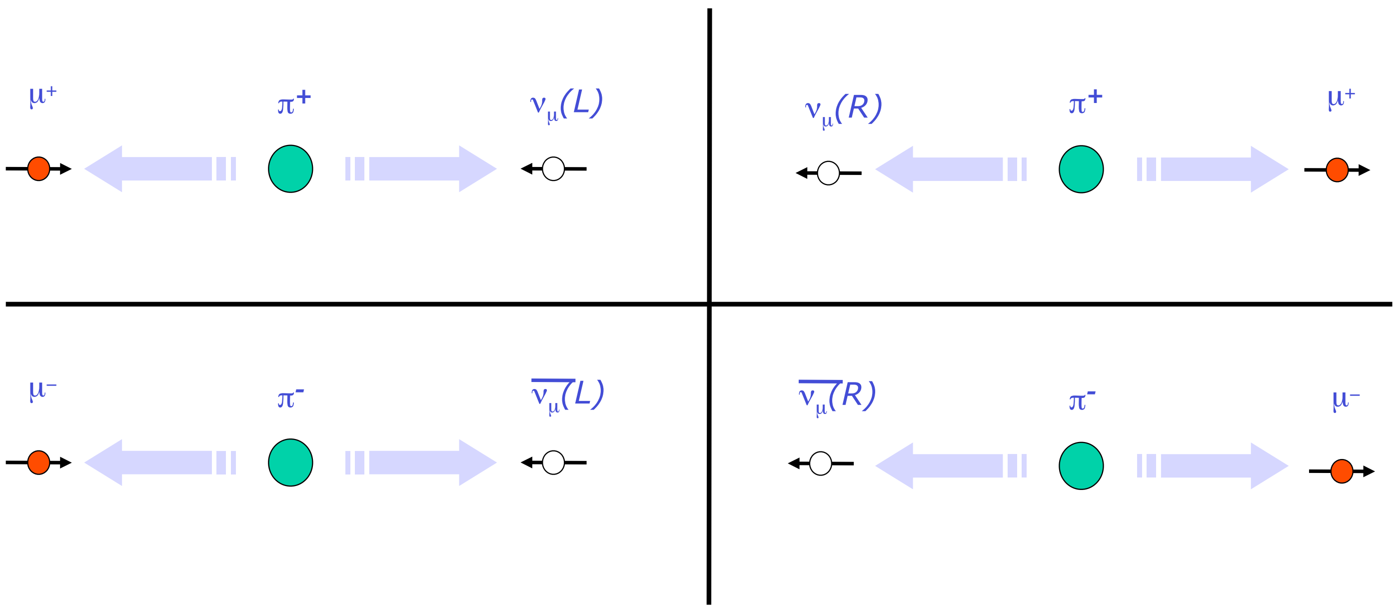


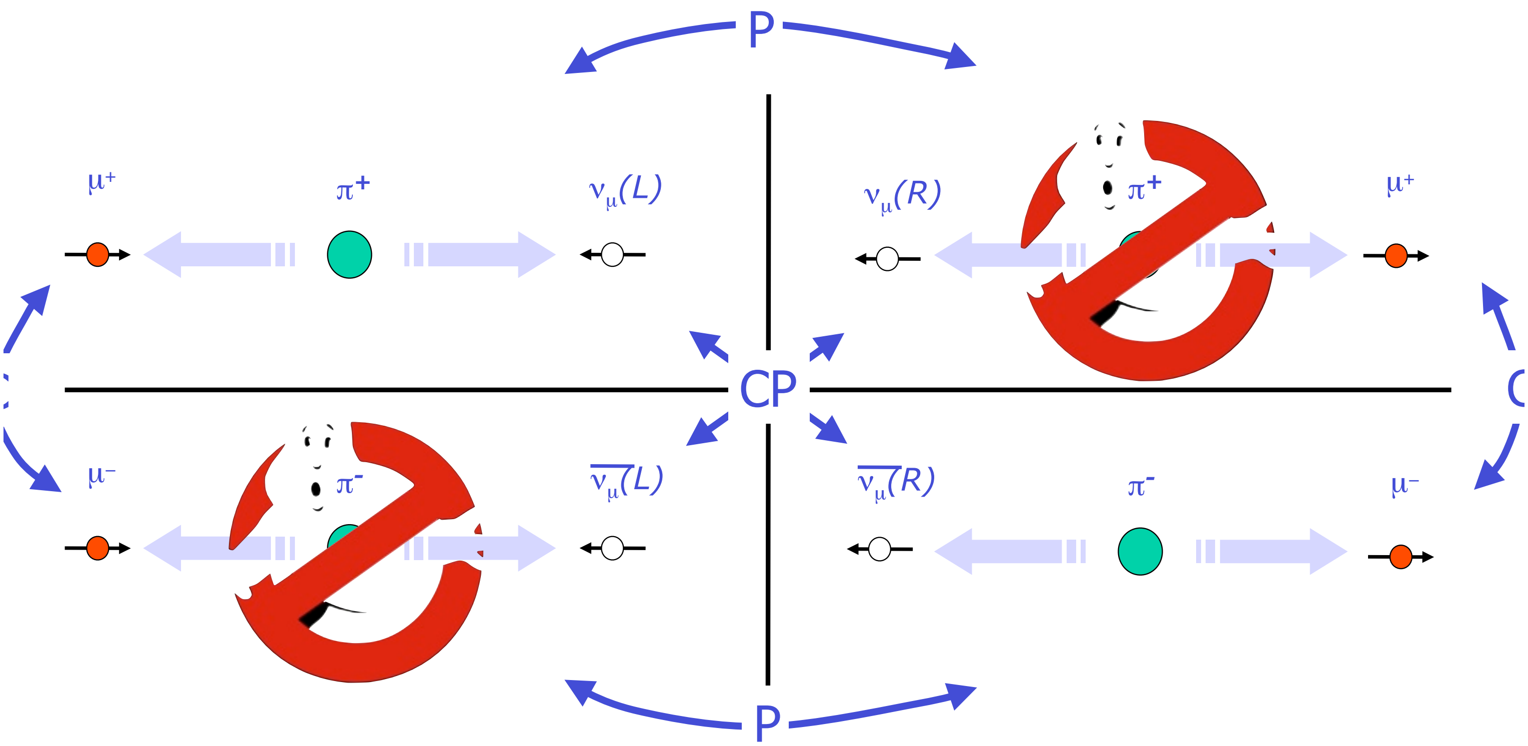
- Nice bonus: can also measure polarization of both neutrino (π^+ decay) and anti-neutrino (π^- decay)

C,P and CP



μ^+
π^+
$\nu_\mu(L)$
$\nu_\mu(R)$
π^+
μ^+
μ^-
π^-
$\overline{\nu_\mu}(L)$
$\overline{\nu_\mu}(R)$
π^-
μ^-

C,P and CP



C broken, P broken, but CP *appears* to
be preserved in weak interaction!

Neutral kaons

Behavior of Neutral Particles under Charge Conjugation

M. Gell-Mann,* *Department of Physics, Columbia University, New York, New York*

AND

A. Pais, *Institute for Advanced Study, Princeton, New Jersey*

(Received November 1, 1954)

Some properties are discussed of the K^0, a heavy boson that is known to decay by the process $K^0 \rightarrow \pi^+ + \pi^-$. According to certain schemes proposed for the interpretation of hyperons and K particles, the K^0 possesses an antiparticle $\bar{K}^0$ distinct from itself. Some theoretical implications of this situation are discussed with special reference to charge conjugation invariance. The application of such invariance in familiar instances is surveyed in Sec. I. It is then shown in Sec. II that, within the framework of the tentative schemes under consideration, the K^0 must be considered as a "particle mixture" exhibiting two distinct lifetimes, that each lifetime is associated with a different set of decay modes, and that no more than half of all K^0's undergo the familiar decay into two pions. Some experimental consequences of this picture are mentioned.

Known:
- $K^0 \rightarrow \pi^+\pi^-$

Hypothesis:
- $\overline{K^0}$ is *not* equal to K^0

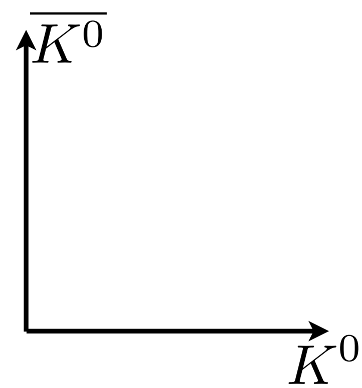
Use C (actually, CP) to deduce:
1. K^0 ($\overline{K^0}$) is an 'admixture' with two distinct lifetimes
2. Each lifetime associated to a distinct set of decay modes
3. No more than 50% of K^0 will decay to two pions...

Neutral Meson Mixing

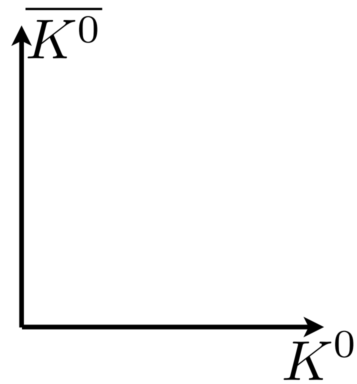
$$\Psi(t) = a(t)\left|K^0\right\rangle + b(t)\left|\overline{K^0}\right\rangle \equiv \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

$$i\frac{\partial}{\partial t}\Psi = \hat{H}\Psi$$

$$\hat{H} = \begin{pmatrix} M_K & 0 \\ 0 & M_K \end{pmatrix}$$



Neutral Meson Mixing



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$$i\frac{\partial}{\partial t}\Psi = \hat{H}\Psi$$

$$\hat{H} = \begin{pmatrix} M_K & 0 \\ 0 & M_K \end{pmatrix}$$

As (eventually) K^0 and $\overline{K^0}$ decay, add an *antihermitic* part to the Hamiltonian

$$\hat{H} = \begin{pmatrix} M_K - \frac{i}{2}\Gamma_K & 0 \\ 0 & M_K - \frac{i}{2}\Gamma_K \end{pmatrix}$$

$$\frac{d}{dt}\left(|a|^2 + |b|^2\right) = -\begin{pmatrix} a^* & b^* \end{pmatrix}\begin{pmatrix} \Gamma_K & 0 \\ 0 & \Gamma_K \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix}$$

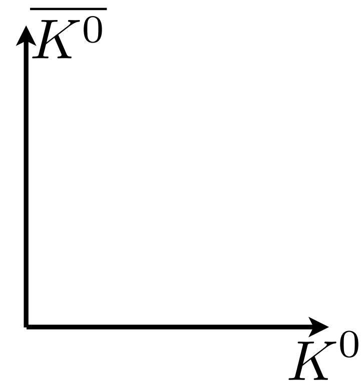
Can identify Γ_K as the decay width ($=1/\tau_K$)

Neutral Meson Mixing

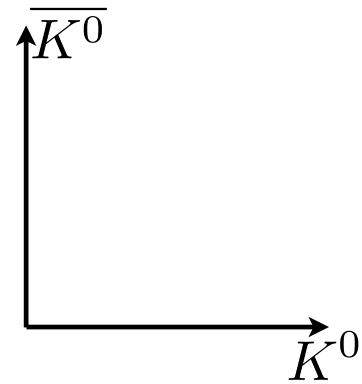
$$\Psi(t) = a(t)\left|K^0\right\rangle + b(t)\left|\overline{K^0}\right\rangle \equiv \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

$$i\frac{\partial}{\partial t}\Psi = \hat{H}\Psi$$

$$\hat{H} = \begin{pmatrix} M_K - \frac{i}{2}\Gamma_K & 0 \\ 0 & M_K - \frac{i}{2}\Gamma_K \end{pmatrix}$$



Neutral Meson Mixing



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low consider the effect of CP symmetry:

CP $\begin{array}{ccc} K^0 & \leftrightarrow & \pi^+\pi^- \\ \overline{K^0} & \leftrightarrow & \pi^+\pi^- \end{array} \Longleftrightarrow K^0 \leftrightarrow \overline{K^0}$

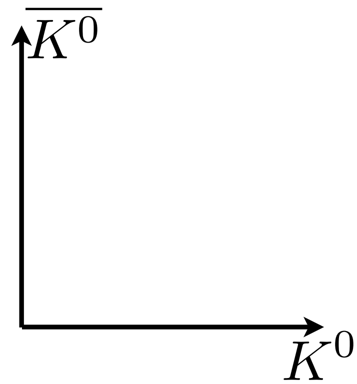
$$\hat{H} = \begin{pmatrix} M_K - \frac{i}{2}\Gamma_K & \Delta \\ \Delta & M_K - \frac{i}{2}\Gamma_K \end{pmatrix}$$

Neutral Meson Mixing

$$\Psi(t) = a(t)\left|K^0\right\rangle + b(t)\left|\overline{K^0}\right\rangle \equiv \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

$$i\frac{\partial}{\partial t}\Psi = \hat{H}\Psi$$

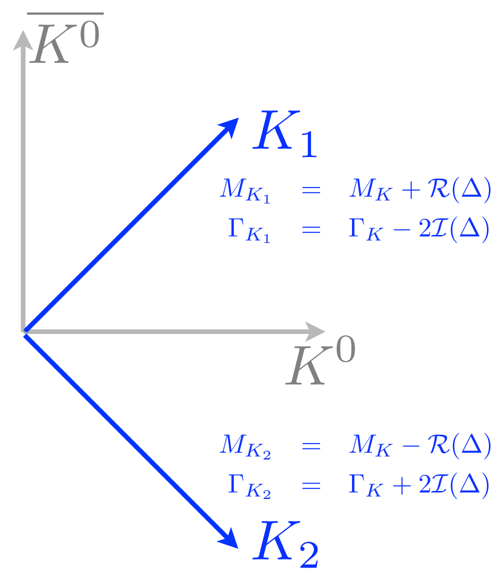
$$\hat{H} = \begin{pmatrix} M_K - \frac{i}{2}\Gamma_K & 0 \\ 0 & M_K - \frac{i}{2}\Gamma_K \end{pmatrix}$$



Now consider the effect of CP symmetry:

$$\mathrm{CP}\quad \begin{array}{ccc} K^0 & \leftrightarrow & \pi^+\pi^- \\ \overline{K^0} & \leftrightarrow & \pi^+\pi^- \end{array} \quad \Leftrightarrow \quad \boxed{K^0 \leftrightarrow \overline{K^0}}$$

$$\hat{H} = \begin{pmatrix} M_K - \frac{i}{2}\Gamma_K & \boxed{\Delta} \\ \boxed{\Delta} & M_K - \frac{i}{2}\Gamma_K \end{pmatrix}$$



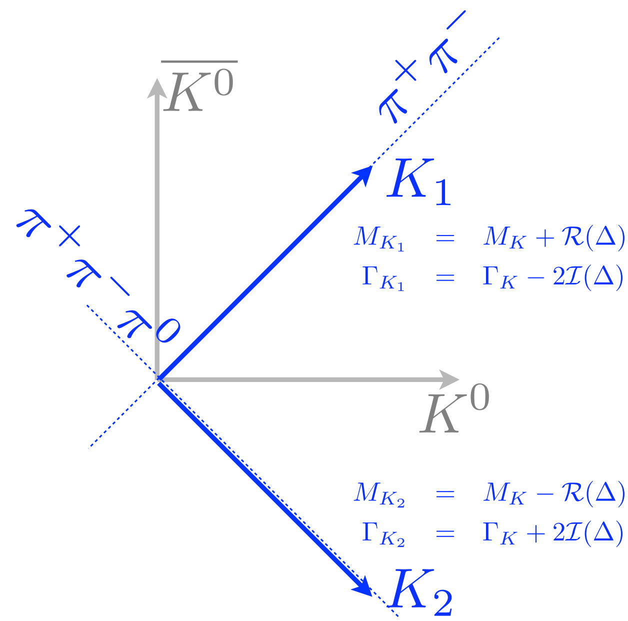
K^0 and $\overline{K^0}$ are *no longer* eigenstates of H
their *sum* (K_1) & *difference* (K_2) are eigenstates...
and K_1 and K_2 have *different* masses and lifetimes

Neutral Kaon Mixing

$$|K_1\rangle = \frac{\left|K^0\right\rangle + \left|\overline{K^0}\right\rangle}{\sqrt{2}}$$

$$|K_2\rangle = \frac{\left|K^0\right\rangle - \left|\overline{K^0}\right\rangle}{\sqrt{2}}$$

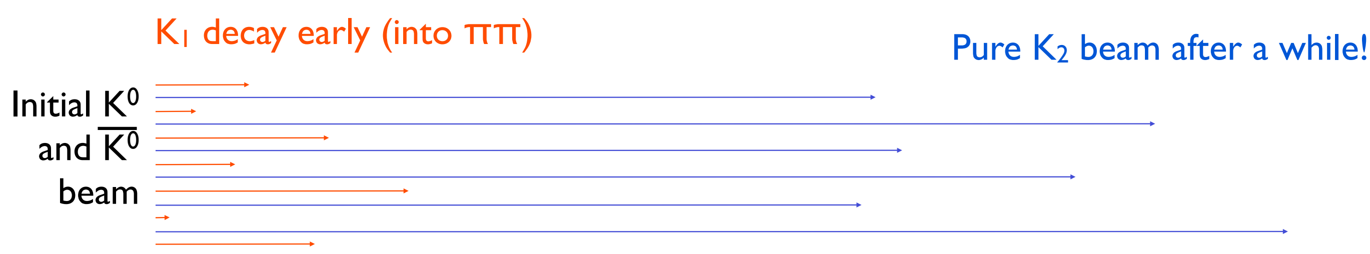
- K_1 and K_2 are their *own* antiparticle, but one is CP even, the other CP odd
- *Only* the CP even state can decay into 2 pions
  - $|K_1\rangle$ (CP=+1) → $\pi\pi$ (CP = -1 * -1 = +1)
- The CP odd state will decay into 3 pions instead
  - $|K_2\rangle$ (CP=-1) → $\pi\pi\,\pi$ (CP = -1*-1*-1 = -1)
- <u>There is a *huge* difference in available phasespace between the two (~600x!) → the CP *even* state will decay much faster</u>
  - Difference due to $M(K^0) \cong 3M(\pi)$
  - Δ has a large imaginary component!



CP violation

Designing a CP violation experiment

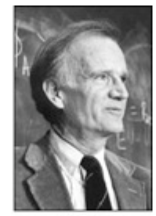
- How do you obtain a pure 'beam' of (CP-odd!) K_2 particles?
- Exploit that decay of K_1 into two pions is *much* faster than decay of K_2 into three pions
  - $\tau_1 = 0.89 \times 10^{-10}$ sec
  - $\tau_2 = 5.2 \times 10^{-8}$ sec (~600 times larger!)
- Beam of neutral Kaons automatically becomes beam of $|K_2\rangle$ as all $|K_1\rangle$ decay very early on…



The Cronin & Fitch Experiment

Essential idea: Look for (CP violating) $K_2 \rightarrow \pi^+\pi^-$ decays 20 meters away from K^0 production point

James Cronin

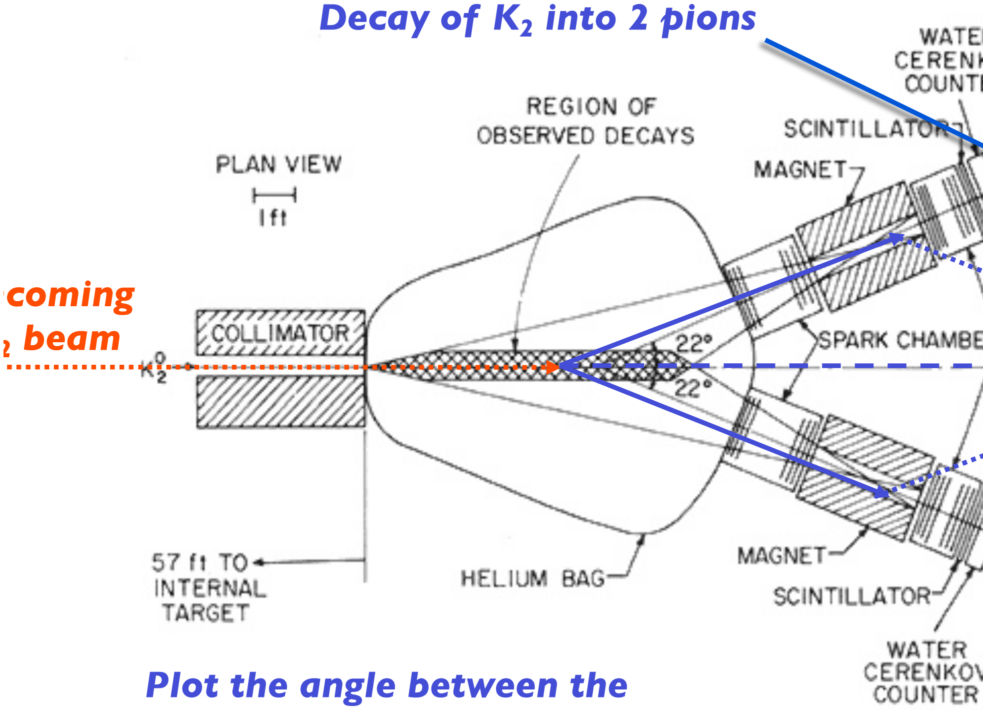


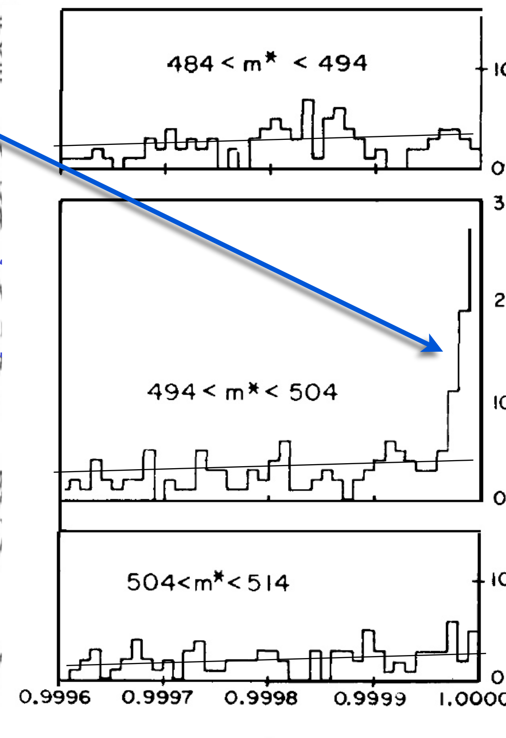
Val Fitch

EVIDENCE FOR THE 2π DECAY OF THE K_2^0 MESON*†

J. H. Christenson, J. W. Cronin,‡ V. L. Fitch,‡ and R. Turla
Princeton University, Princeton, New Jersey
(Received 10 July 1964)

Decay of K_2 into 2 pions



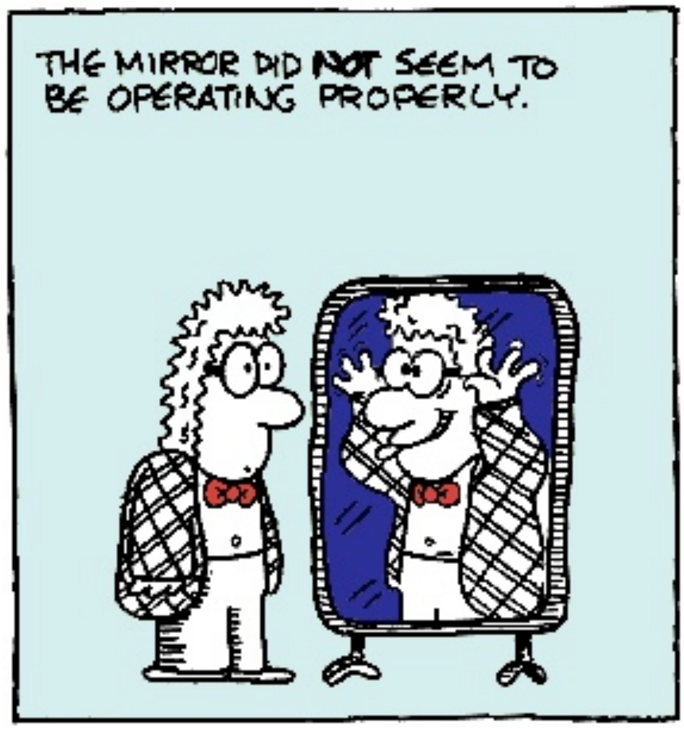


Plot the angle between the momentum direction of two

CP is 'a bit' broken by weak interaction

Nobel prize 1980:

"The discovery emphasizes, once again, that even almost self evident principles in science cannot be regarded fully valid until they have been critically examined in precise experiments."



How to construct a physics law that violates a symmetry just a tiny bit?

- Only 0.2% of K_2 decays violate CP...
- Maximal (100%) violation of P symmetry "easily" interpretable/explained as absence of a right-handed neutrino...

Baryon asymmetry of the Universe

Observed matter-antimatter asymmetry

- **Main questions:** Why do the Earth, the Solar system and our galaxy consists of of matter and not of antimatter?
- Why we do not see any traces of antimatter in the universe except of those where antiparticles are created in collisions of ordinary particles?
- This looks really strange, as the properties of matter and antimatter are very similar.

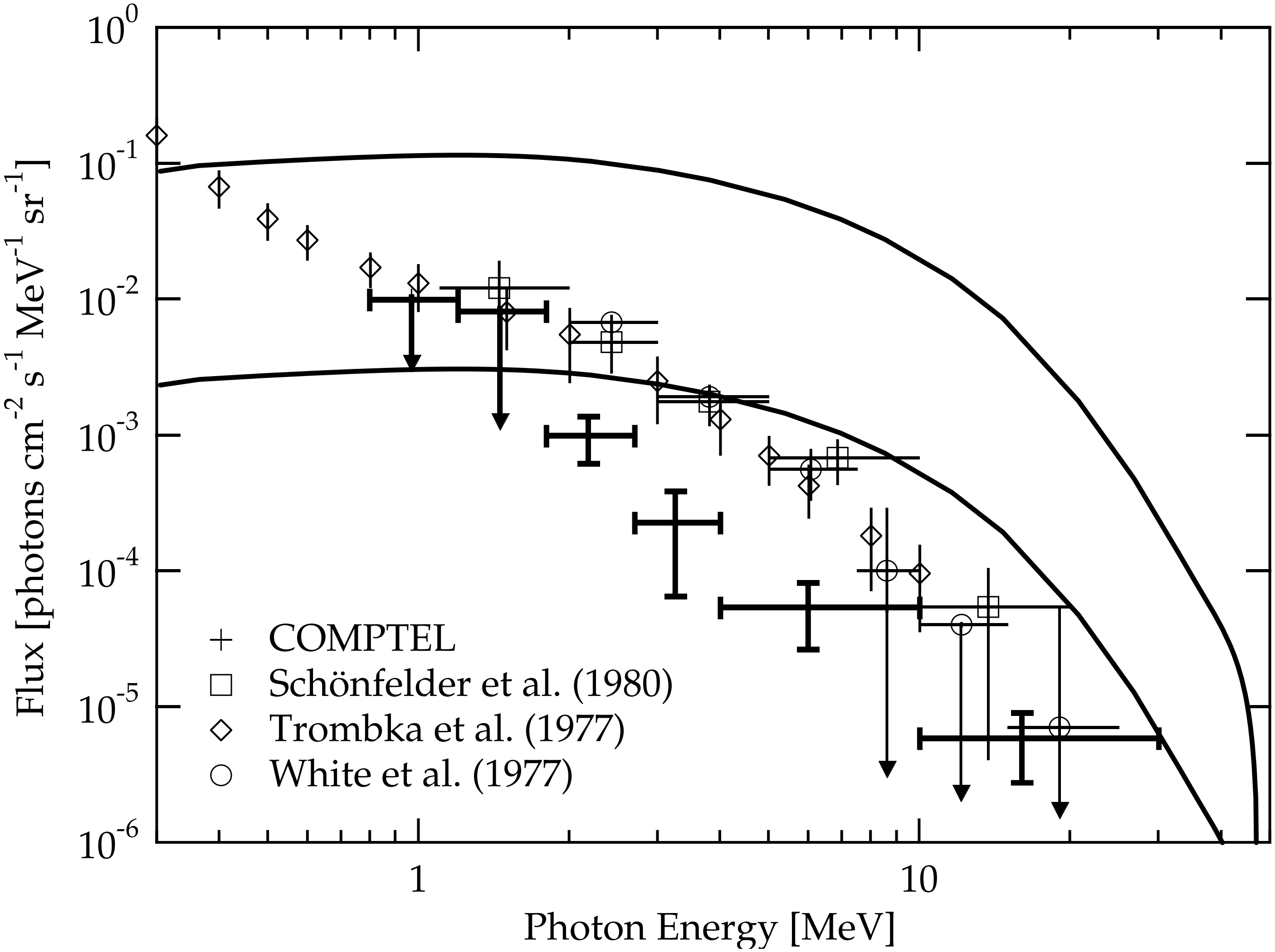
There are two possibilities:

- Observed universe is asymmetric and does not contain any antimatter

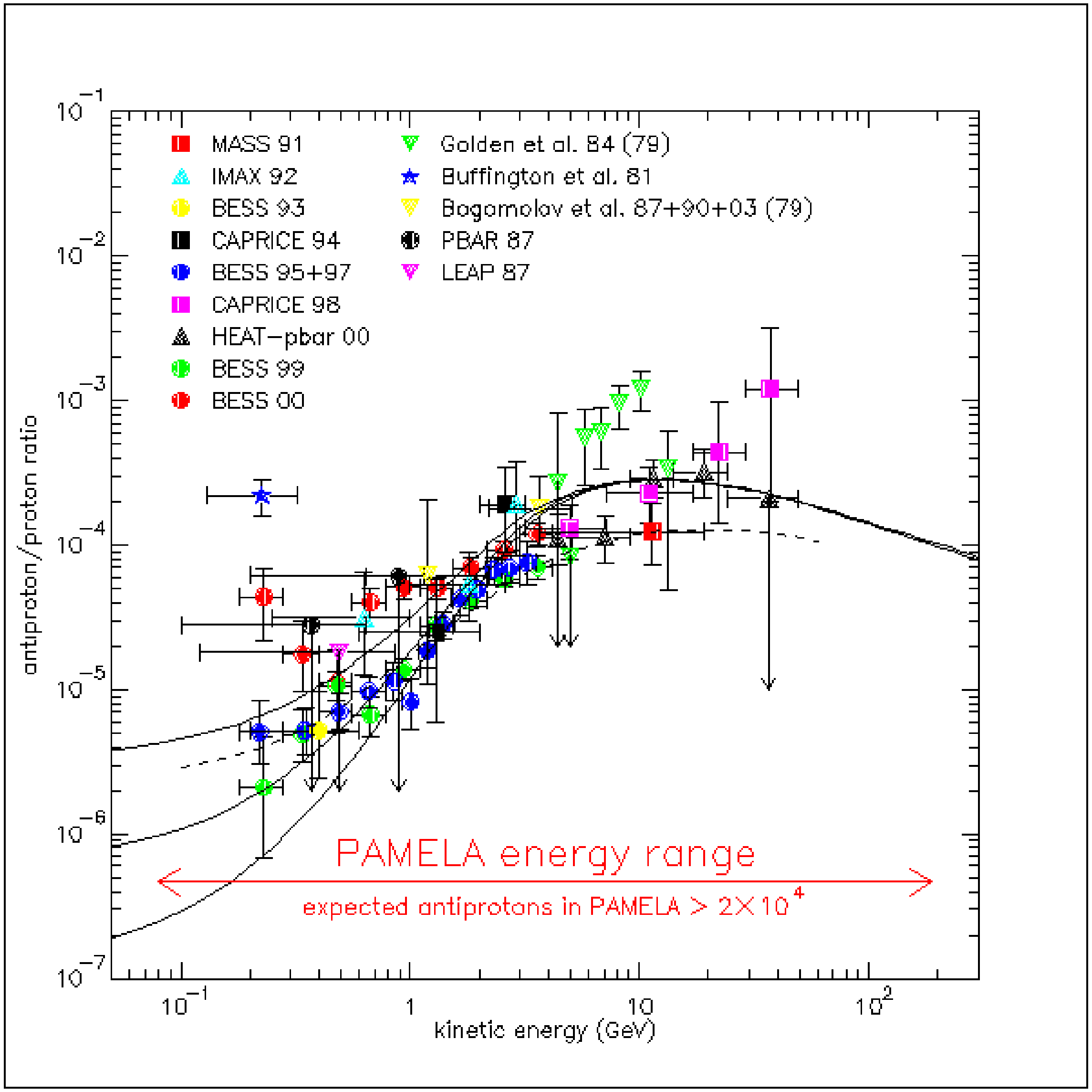
- The universe consists of domains of matter and antimatter separated by voids to prevent annihilation. The size of these zones should be greater than 1000 Mpc, in order not to contradict observations of the diffuse γ spectrum.

The second option, however, contradicts to the large scale isotropy of the cosmic microwave background.

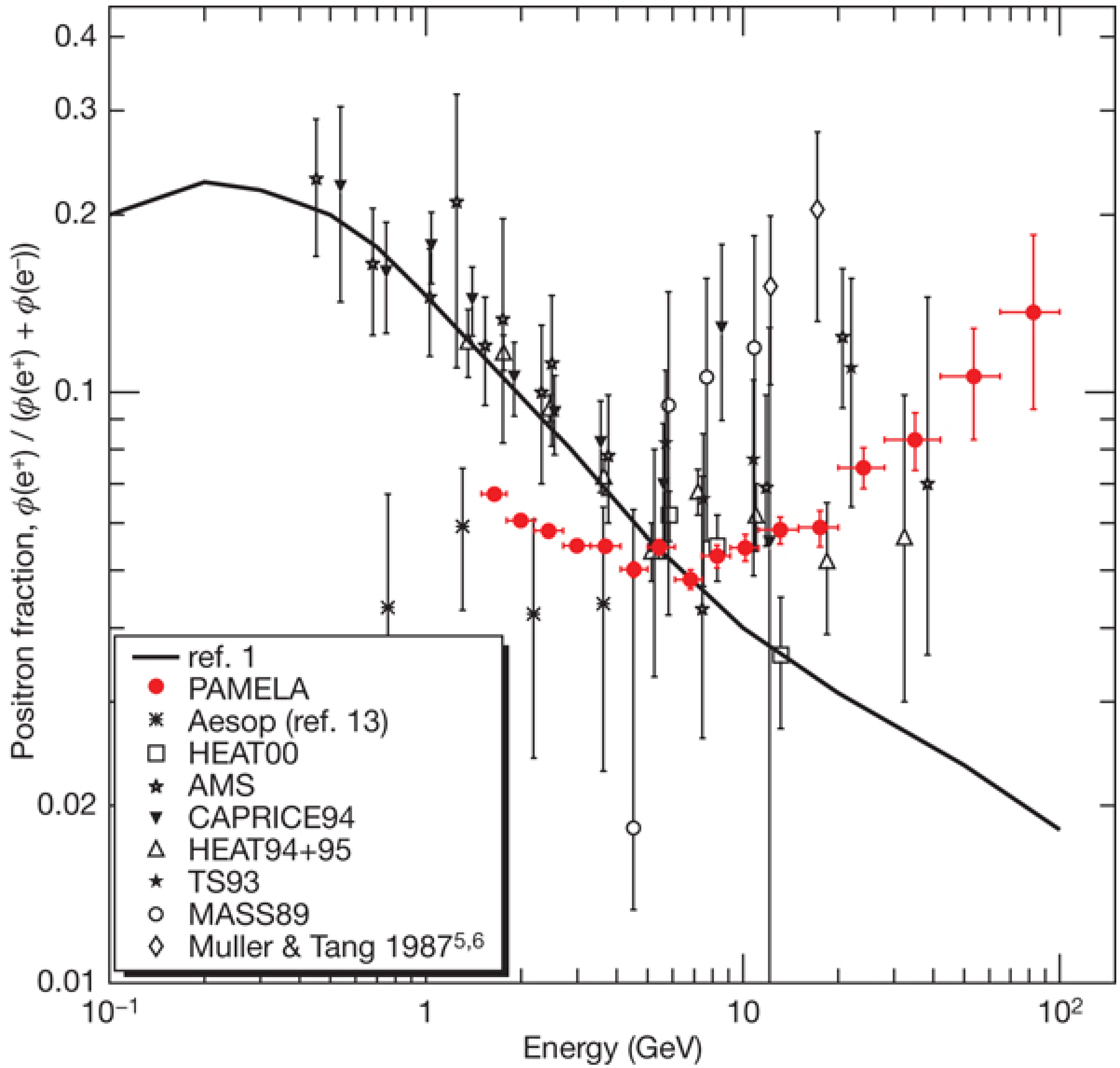
Thus, we are facing the question: Why the universe is globally asymmetric?



Data and expectations for the diffuse γ-ray spectrum (upper curve d = 20 Mpc, lower curve d = 1000 Mpc)



Example: antiproton-to-proton fraction in GeV: $10^{-7} - 10^{-3}$

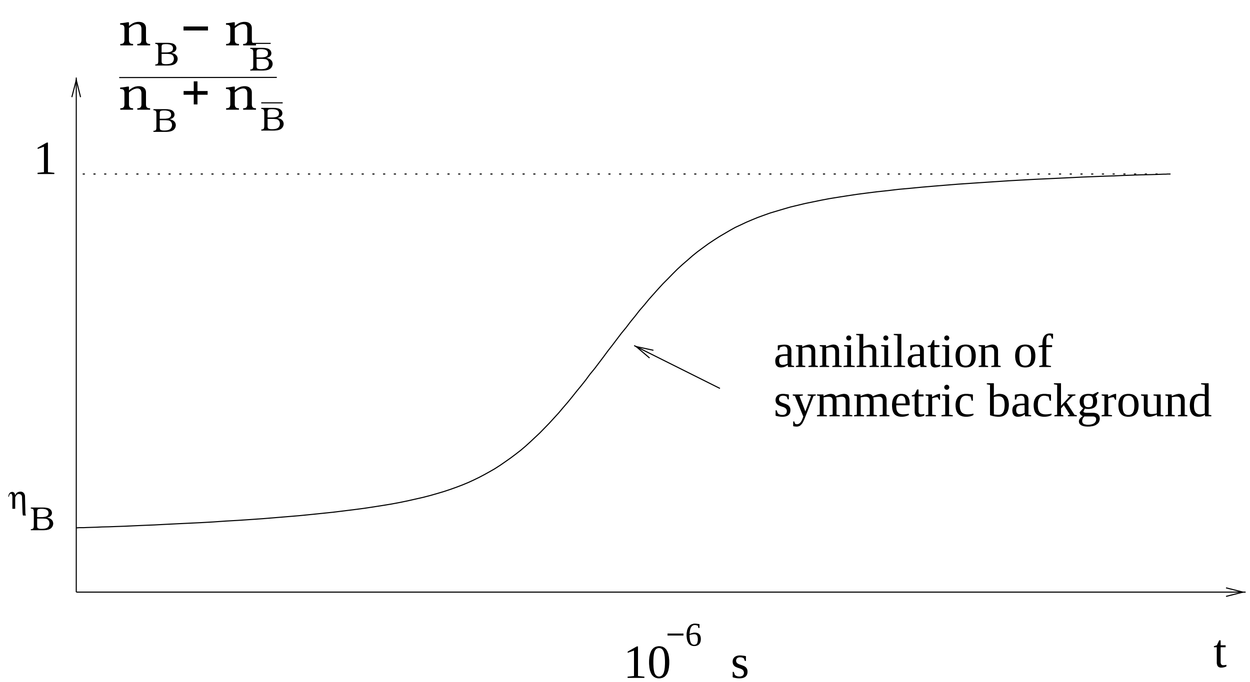


Example: positron-to-electron fraction in GeV: $0.02 - 0.2$

IF THERE ARE NO ANTI-BARYONS NOW, WHAT KIND OF ASYMMETRY THIS IMPLIES IN THE EARLY UNIVERSE?

Thermal history of baryon asymmetry

- The baryon asymmetry $\boxed{\Delta_B = \frac{n_B - n_{\bar{B}}}{n_B + n_{\bar{B}}}}$ **today** is very close to 1
- How did it evolve? What was its value in the earlier times?
- Any quark (actually, any particle) is present in the plasma if $T \gtrsim m_q$, because of the annihilation processes $q + \bar{q} \leftrightarrows \gamma + \gamma$ (or $q + \bar{q} \leftrightarrows g + g$)



Thermal history of baryon asymmetry

- At early times for each 10^{10} quarks there is $10^{10} - \mathbf{1}$ antiquark.

- Symmetric quark-antiquark background annihilates into photons and neutrinos while the asymmetric part survives and gives rise to galaxies, stars, planets.

Sakharov's conditions on the Big Bang

OLATION OF CP INVARIANCE, C ASYMMETRY, AND BARYON ASYMMETRY OF THE UNIVERSE

A. D. Sakharov
Submitted 23 September 1966
ZhETF Pis'ma 5, No. 1, 32-35, 1 January 1967

The theory of the expanding Universe, which presupposes a superdense initial state of tter, apparently excludes the possibility of macroscopic separation of matter from antitter; it must therefore be assumed that there are no antimatter bodies in nature, i.e., the iverse is asymmetrical with respect to the number of particles and antiparticles asymmetry). In particular, the absence of antibaryons and the proposed absence of ryonic neutrinos implies a non-zero baryon charge (baryonic asymmetry). We wish to point t a possible explanation of C asymmetry in the hot model of the expanding Universe (see [1]) making use of effects of CP invariance violation (see [2]). To explain baryon asymmetry, propose in addition an approximate character for the baryon conservation law.

Andrei Sakharov
"Father" of Soviet hydrogen bomb
& Nobel Peace Prize Winner

Three requirements for a universe with a baryon asymmetry:

1. A process that violates baryon number
2. C and CP violation, i.e. breaking of the C and CP *symmetries*
3. 1 & 2 should occur during a phase which is NOT in thermal equilibrium

Sakharov conditions-I

- **If baryon number is conserved**, then in every process
$$\psi_1 + \psi_2 + \cdots \to \chi_1 + \chi_2 + \ldots$$
left hand side and right hand side contain equal number of (baryons - anti-baryons)

- Experimentally we see that baryon charge is conserved in particle physics processes.

- As a consequence proton (the latest baryon) is stable (proton lifetime $> 6.6 \times 10^{33}$ years for decays such as $p \to \pi^0 + e^+$ or $p \to \pi^0 + \mu^+$. This bound is 5×10^{23} times longer than the age of the Universe

- The conservation of baryon number would mean that the total baryon charge of the Universe remains constant in the process of evolution.

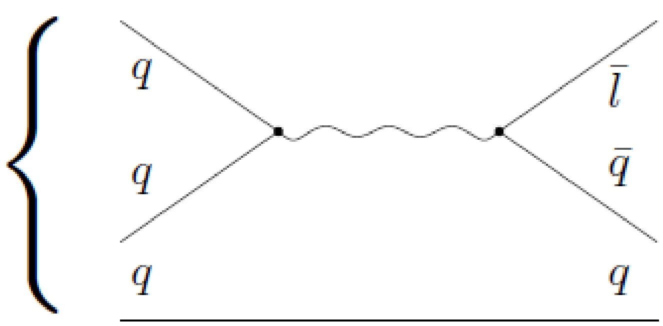
- **If** initial conditions were matter-antimater symmetric – no baryon asymmetry could have been generated

Problem. Taking the lower bound on the proton's lifetime as its actual lifetime, compute the amount of water one would need to observe 1 proton decay during 10 years.

- Postulate new interaction mediated but a massive gauge boson (**X-boson**), transforming quarks to leptons: $X \to q + \ell$ (similar to W boson in electroweak theory where $W \to e + \bar{\nu}_e$)

- As a consequence, the processes with X-boson exchange **violate** the baryon number

- For example, the protons may **decay**.



The proton decays mediated by X-boson:

$\Rightarrow \quad p \to e^{+} + \pi^{0}$

$\Rightarrow \quad p \to \bar{\nu}_e + \pi^{+}$

- The proton lifetime can be estimated as (similar to muon decay):

$$\boxed{\tau_p^{-1} \sim \left(\frac{\alpha_X}{M_X^2}\right)^2 m_p^5} \qquad \tau_\mu^{-1} \sim \left(\frac{\alpha_W}{M_W^2}\right)^2 m_\mu^5$$

- Existing experimental bounds on the proton lifetime: $\tau_p \gtrsim 10^{33}$ yrs gives $M_X \gtrsim 10^{16}$ GeV.

- Yukawa couplings may violate CP (**Sakharov conditions**).

- However, this mechanism requires **new physics** at $E \sim M_X \ldots$

Can we generate baryon number at **lower** energies?
YES!

Quantum anomalies

(violation of classical symmetries at quantum level)

- Massless fermions can be left and right-chiral (left and right moving):

$$(i\gamma^\mu\partial_\mu - \cancel{m})\psi = \begin{pmatrix} \cancelto{0}{-m} & i(\partial_t + \vec{\sigma}\cdot\vec{\nabla}) \\ i(\partial_t - \vec{\sigma}\cdot\vec{\nabla}) & \cancelto{0}{-m} \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = 0$$

  where $\gamma_5\psi_{R,L} = \pm\psi_{R,L}$ and $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$

- Two global symmetries: $\psi_L \to e^{i\alpha}\psi_L$ and $\psi_R \to e^{i\beta}\psi_R$

- According to the Nöther theorem, one can define two **independently** conserved charges that we call the number of left-movers $N_L = \int d^3x\, \psi_L^\dagger\psi_L$ and the number of right-movers $N_R = \int d^3x\, \psi_R^\dagger\psi_R$.

  Linear combinations of these charges are known as **fermion number** $N_L + N_R$ (current $\bar\psi\gamma^\mu\psi$) and **axial fermion number** $N_L - N_R$ (current $\bar\psi\gamma^\mu\gamma_5\psi$). Again, both are conserved independently in the free theory

- Gauge interactions respects chirality ($D_\mu = \partial_\mu + eA_\mu$)...

$$\begin{pmatrix} 0 & i(D_t + \vec{\sigma}\cdot\vec{D}) \\ i(D_t - \vec{\sigma}\cdot\vec{D}) & 0 \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = 0$$

The symmetry $\psi \to e^{i\alpha(x)}\psi$ is *gauged*, but the Lagrangian seemingly still preserves both global symmetries: $\psi_L \to e^{i\alpha}\psi_L$ and $\psi_R \to e^{i\beta}\psi_R$

- ...but the difference of left and right-movers is **not conserved anymore**:

$$\frac{d(N_L - N_R)}{dt} = \int d^3\vec{x}\left(\partial_\mu j^5_\mu\right) = \frac{e^2}{2\pi^2}\int d^3\vec{x}\,\vec{E}\cdot\vec{B} \neq 0$$

...if one takes into account quantum corrections

Reminder: Landau levels

- Recall: particle in the uniform magnetic field , parallel to z axis: $\vec{B} = (0, 0, B)$
- Take Dirac equation coupled to the gauge potential $\vec{A} = (0, Bx, 0)$
- Conserved quantities: energy, momenta p_y, p_z
- Take the square of the Dirac equation to get:

$$\left(-\frac{d^2}{dx^2} + (eBx - p_y)^2 - 2eBs_z\right)\phi = (E^2 - p_z^2)\phi \tag{9}$$

Spin projection $s_z = \pm\frac{1}{2}$

- The l.h.s. of (9) is just a Schödinger equation for the harmonic oscillator with the frequency $\omega = 2eB$, whose origin is shifted by $\pm eB$
- The energy levels of harmonic oscillators $\epsilon_n = \omega(n + \frac{1}{2})$, $n \geq 0$

- Therefore, the spectrum of Eq. (9) is given by

$$E_n^2 - p_z^2 = eB(2n+1) + 2s_z eB \qquad (10)$$

- Spectrum has three quantum numbers:

  ▷ $n = 0, 1, 2 \ldots$
  ▷ $-\infty \le p_z \le +\infty$
  ▷ $s_z = \pm\frac{1}{2}$

- Consider $n = 0$. For $s_z = -\frac{1}{2}$ the spectrum (10) becomes

$$E^2 = p_z^2 \qquad \text{massless 1-dimensional fermion} \qquad (11)$$

  for $s_z = +\frac{1}{2}$ there is **no** massless mode

- For $n > 0$ there is no cancellation between $eB(2n+1)$ and $2s_z eB$ term

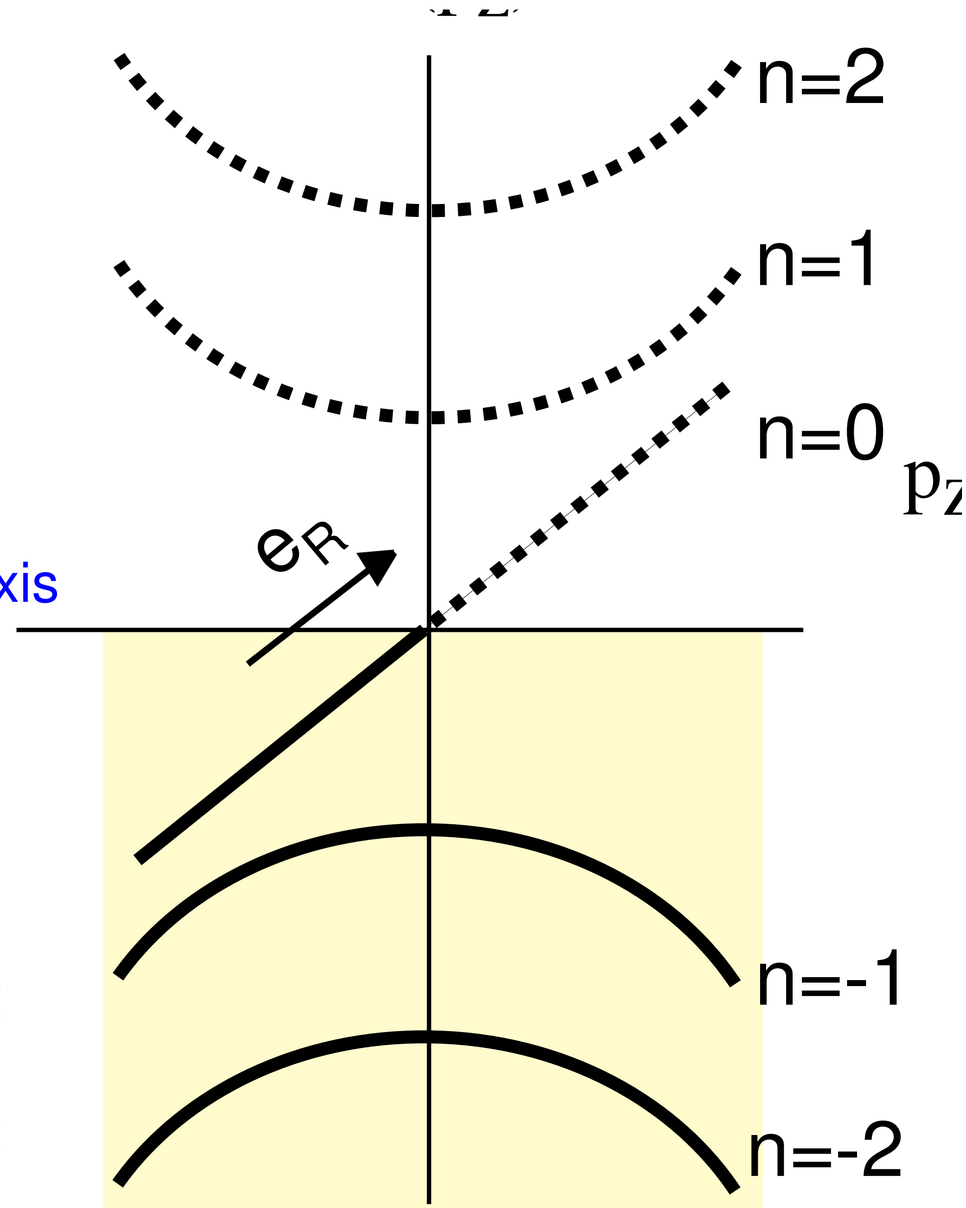
- Consider Landau levels:

$$E^2 = p_z^2 + eB(2n+1) + 2e\vec{B}\cdot\vec{s}$$

- Particles with $\vec{B}\cdot\vec{s} < 0$ have **massless** branches:

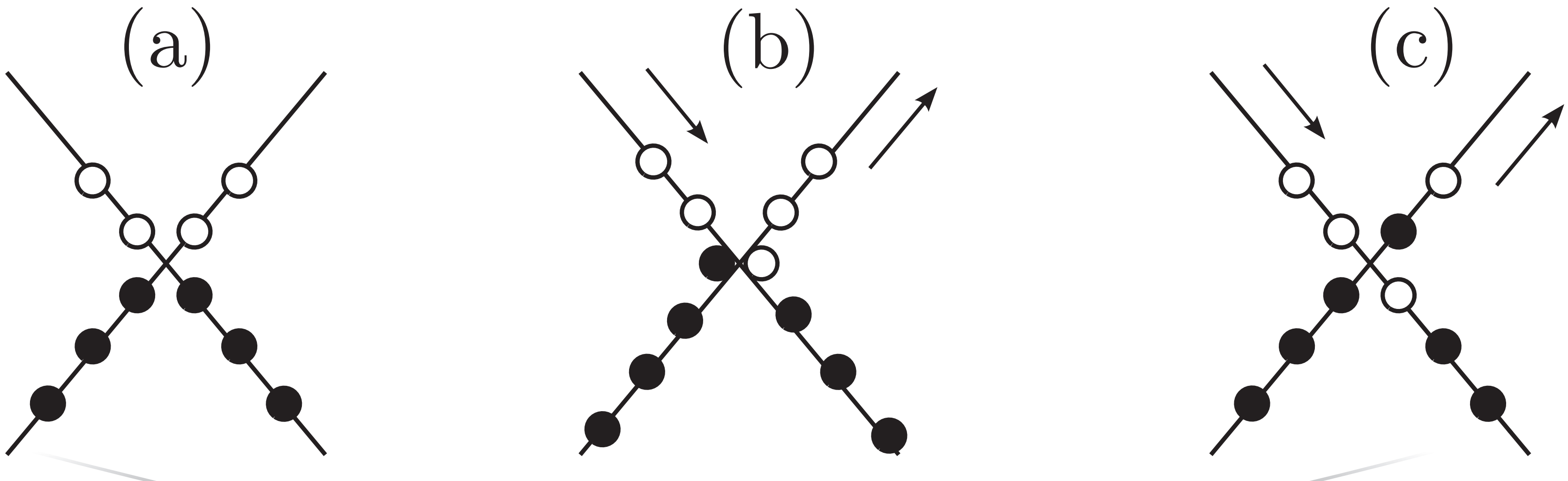
$$E = \begin{cases} -p_z & \text{move down along z-axis} \\ p_z & \text{move up along z-axis} \end{cases}$$

- Dirac vacuum $\leftrightarrow$ all states $E < 0$ are filled:
  - ▷ $E = -p_z < 0 \Rightarrow \vec{p}\cdot\vec{s} < 0$ – left particles
  - ▷ $E = p_z < 0 \Rightarrow \vec{p}\cdot\vec{s} > 0$ – right particles



Chiral anomaly explained

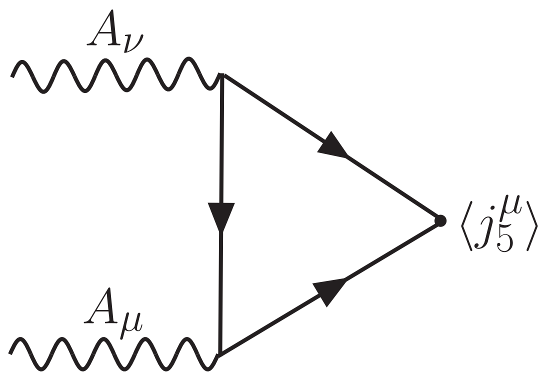
- Electric field $\vec{E} = E\hat{z}$ **creates** right particle (because $p_z(t) = p_z(0) + eEt$)
- For particles of the other chirality the situation is opposite: such electric field destroys such particles (creates **hole** in the Dirac sea)



- As a result:
  - **Total number** of **particles** does not change
  - **Difference** of left minus right appears – **chiral anomaly!**
- This only happens when $\vec{E} \parallel \vec{B}$ (proportional to $\vec{E}\cdot\vec{B}$)

- One can see this for example by computing the diagram with two electromagnetic vertices and one axial current

Adler, Bell, Jackiw



- This diagram changes sign if we change all **left particles** into **right particles** (Furry theorem). Therefore it is proportional to

$$\text{triangular diagram} = \left[e^2(-1) - e^2(+1)\right] F(p, \epsilon_\mu, \epsilon_\nu)$$

- In general, any triangular diagram factorizes into some momentum integral, multiplied by

$$\textbf{Anomaly cancellation condition} = \boxed{\left(\sum_{left} e_L^2 Q_L - \sum_{right} e_R^2 Q_R\right)}$$

where e_L, e_R are **gauge charges** of left/right particles and Q_L, Q_R are charges with respect to the **global symmetry**

- **Anomaly** means that the notion of left and right chirality for massless fermions is not **gauge invariant**
- What if current in the vertex is the gauge current? Anomaly of a **gauge symmetry** renders theory inconsistent (non-unitary):

$$\partial_\nu\left(\partial_\mu F^{\mu\nu} = j^\nu\right) \Longrightarrow 0 = \partial_\nu j^\nu$$

- Gauge anomaly cancels if charges of “vector-like” (for each fermion $e_L = e_R$). For example, electrodynamic is vector like.

- Consider two Dirac fermions $b = (b_L, b_R)$ and $\ell = (\ell_L, \ell_R)$, charged with respect to a gauge group U(1) in the following way:

$$-Q(b_L) = Q(\ell_L) = e$$
$$Q(b_R) = Q(\ell_R) = 0$$

- The Lagrangian has the form

$$\mathcal{L} = \bar{b}\Big(\not{\partial} - \frac{e}{2}\not{A}(1-\gamma_5)\Big)b + m_b\bar{b}b + \bar{\ell}\Big(\not{\partial} + \frac{e}{2}\not{A}(1-\gamma_5)\Big)\ell + m_\ell\bar{\ell}\ell$$

  – U(1) theory with **chiral gauge group**

- This theory does not have gauge anomaly, as “anomaly cancellation condition” is satisfied:

$$Q(b_L)^3 + Q(\ell_L)^3 - Q(b_R)^3 - Q(\ell_R)^3 = 0$$

- At classical level the theory has two **global** U(1) symmetries:

$$b \to e^{i\alpha} b \quad \text{and} \quad \ell \to e^{i\beta}\ell$$

with the corresponding Nöther currents:

$$J_b^\mu = \bar{b}\gamma^\mu b \quad ; \quad J_\ell^\mu = \bar{\ell}\gamma^\mu \ell$$

you can think of corresponding conserved charges as analog of **baryon** and **lepton** number

- Are these symmetries anomalous?

- Compute anomaly cancellation condition for J_b^μ:

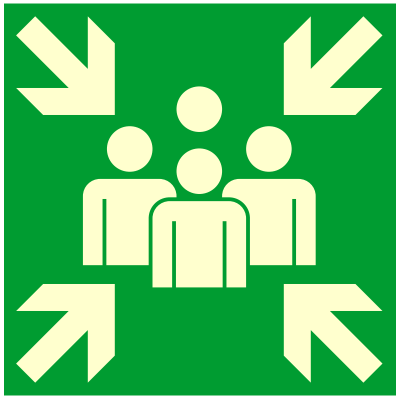
$$\underbrace{(-e)^2(+1)}_{b_L} + \underbrace{e^2 \times 0}_{\ell_L} - \underbrace{0^2 \times (+1)}_{b_R} - \underbrace{0^2 \times 0}_{\ell_R} = -e^2 \neq 0$$

$\Rightarrow$ symmetry J_b^μ is anomalous (same is true for J_ℓ^μ)

U(1) model

In the theory with vector-like symmetries **but** chiral gauge charges these vector-like symmetries are anomalous!
(if **both** gauge interactions **and** symmetries are vector-like $\Rightarrow$ there is no anomaly)

To summarize: quantum anomaly



- Notion of chirality (left/right particles) may get incompatible with requirement of gauge invariants in **quantum** theories.
- In this case the chirality (or *the number of left minus right particles*) changes if one turns out field configurations proportional to $\vec{E} \cdot \vec{B}$.
- Anomalies in general appear when there is
  - ▷ Chiral current in the background of vector-like gauge fields
  - ▷ Vector-like current in the background of chiral gauge fields
  - ▷ Chiral current in the background of chiral gauge fields

Standard Model of

FUNDAMENTAL PARTICLES AND INTERACTIONS

The Standard Model summarizes the current knowledge in Particle Physics. It is the quantum theory that includes the theory of strong interactions (quantum chromodynamics or QCD) and the unified theory of weak and electromagnetic interactions (electroweak). Gravity is included on this chart because it is one of the fundamental interactions even though not part of the "Standard Model."

FERMIONS matter constituents spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2

Flavor	Mass GeV/c^2	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0
e electron	0.000511	−1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	−1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	−1

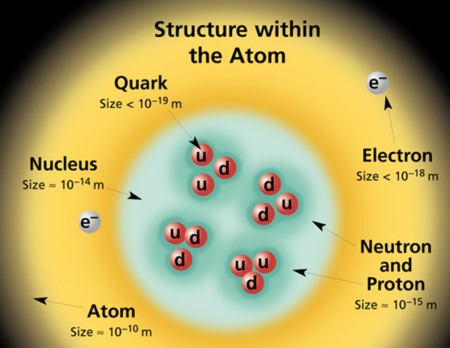
Quarks spin = 1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.003	2/3
d down	0.006	−1/3
c charm	1.3	2/3
s strange	0.1	−1/3
t top	175	2/3
b bottom	4.3	−1/3

Spin is the intrinsic angular momentum of particles. Spin is given in units of ħ, which is the quantum unit of angular momentum, where $\hbar = h/2\pi = 6.58\times10^{-25}$ GeV s $= 1.05\times10^{-34}$ J s.

Electric charges are given in units of the proton's charge. In SI units the electric charge of the proton is 1.60×10^{-19} coulombs.

The **energy** unit of particle physics is the electronvolt (eV), the energy gained by one electron in crossing a potential difference of one volt. **Masses** are given in GeV/c^2 (remember $E = mc^2$), where 1 GeV $= 10^9$ eV $= 1.60\times10^{-10}$ joule. The mass of the proton is 0.938 GeV/c^2 $= 1.67\times10^{-27}$ kg.



If the protons and neutrons in this picture were 10 cm across, then the quarks and electrons would be less than 0.1 mm in size and the entire atom would be about 10 km across.

BOSONS force carriers spin = 0, 1, 2, ...

Unified Electroweak spin = 1

Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.4	−1
W^+	80.4	+1
Z^0	91.187	0

Strong (color) spin = 1

Name	Mass GeV/c^2	Electric charge
g gluon	0	0

Color Charge

Each quark carries one of three types of "strong charge," also called "color charge." These charges have nothing to do with the colors of visible light. There are eight possible types of color charge for gluons. Just as electrically-charged particles interact by exchanging photons, in strong interactions color-charged particles interact by exchanging gluons. Leptons, photons, and ***W*** and ***Z*** bosons have no strong interactions and hence no color charge.

Quarks Confined in Mesons and Baryons

One cannot isolate quarks and gluons; they are confined in color-neutral particles called **hadrons**. This confinement (binding) results from multiple exchanges of gluons among the color-charged constituents. As color-charged particles (quarks and gluons) move apart, the energy in the color-force field between them increases. This energy eventually is converted into additional quark-antiquark pairs (see figure below). The quarks and antiquarks then combine into hadrons; these are the particles seen to emerge. Two types of hadrons have been observed in nature: **mesons** $q\bar{q}$ and **baryons** qqq.

Residual Strong Interaction

The strong binding of color-neutral protons and neutrons to form nuclei is due to residual strong interactions between their color-charged constituents. It is similar to the residual electrical interaction that binds electrically neutral atoms to form molecules. It can also be viewed as the exchange of mesons between the hadrons.

PROPERTIES OF THE INTERACTIONS

Property \ Interaction	Gravitational	Weak (Electroweak)	Electromagnetic (Electroweak)	Strong: Fundamental	Strong: Residual
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge	See Residual Strong Interaction Note
Particles experiencing:	All	Quarks, Leptons	Electrically charged	Quarks, Gluons	Hadrons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons	Mesons
Strength relative to electromag for two u quarks at: 10^{-18} m	10^{-41}	0.8	1	25	Not applicable to quarks
Strength relative to electromag for two u quarks at: 3×10^{-17} m	10^{-41}	10^{-4}	1	60	
for two protons in nucleus	10^{-36}	10^{-7}	1	Not applicable to hadrons	20

Baryons qqq and Antibaryons $\bar{q}\bar{q}\bar{q}$

Baryons are fermionic hadrons. There are about 120 types of baryons.

Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
p	proton	uud	1	0.938	1/2
$\bar{p}$	anti-proton	$\bar{u}\bar{u}\bar{d}$	−1	0.938	1/2
n	neutron	udd	0	0.940	1/2
Λ	lambda	uds	0	1.116	1/2
Ω^-	omega	sss	−1	1.672	3/2

Mesons $q\bar{q}$

Mesons are bosonic hadrons. There are about 140 types of mesons.

Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
π^+	pion	$u\bar{d}$	+1	0.140	0
K^-	kaon	$s\bar{u}$	−1	0.494	0
ρ^+	rho	$u\bar{d}$	+1	0.770	1
B^0	B-zero	$d\bar{b}$	0	5.279	0
η_c	eta-c	$c\bar{c}$	0	2 .980	0

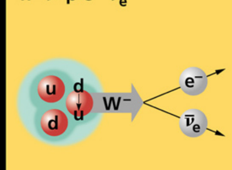
Matter and Antimatter

For every particle type there is a corresponding antiparticle type, denoted by a bar over the particle symbol (unless + or – charge is shown). Particle and antiparticle have identical mass and spin but opposite charges. Some electrically neutral bosons (e.g., Z^0, γ, and $\eta_c = c\bar{c}$, but not $K^0 = d\bar{s}$) are their own antiparticles.

Figures

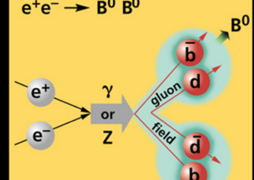
These diagrams are an artist's conception of physical processes. They are *not* exact and have *no* meaningful scale. Green shaded areas represent the cloud of gluons or the gluon field, and red lines the quark paths.

$n \rightarrow p\, e^-\, \bar{\nu}_e$



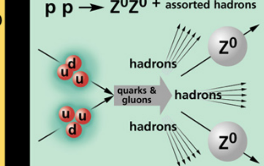
A neutron decays to a proton, an electron, and an antineutrino via a virtual (mediating) W boson. This is neutron β decay.

$e^+e^- \rightarrow B^0\, \bar{B}^0$



An electron and positron (antielectron) colliding at high energy can annihilate to produce B^0 and $\bar{B}^0$ mesons via a virtual Z boson or a virtual photon.

$p\, p \rightarrow Z^0 Z^0$ + assorted hadrons



Two protons colliding at high energy can produce various hadrons plus very high mass particles such as Z bosons. Events such as this one are rare but can yield vital clues to the structure of matter.

The Particle Adventure

Visit the award-winning web feature *The Particle Adventure* at **http://ParticleAdventure.org**

This chart has been made possible by the generous support of:
U.S. Department of Energy
U.S. National Science Foundation
Lawrence Berkeley National Laboratory
Stanford Linear Accelerator Center
American Physical Society, Division of Particles and Fields
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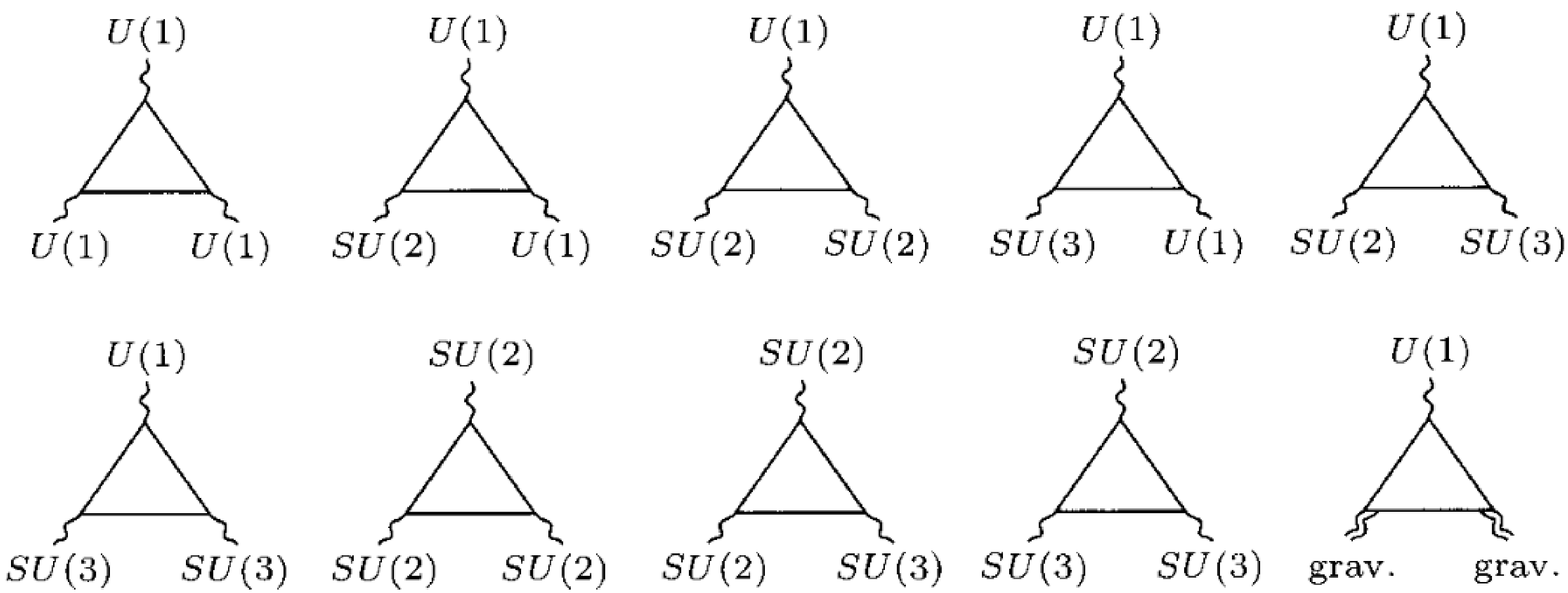
- **BUT!** in the SM electroweak interactions are chiral. If the notion of chirality is **not-gauge invariant** – how SM can be consistent?
- There are many left and many right states in the SM. If you sum them up – they cancel all anomalies



From **Peskin & Schroeder [Sec. 20.2]**

Anomalous fermion numbers non-conservation

- Baryon number in the Standard Model:

$$J_B^\mu = \frac{1}{3}(\bar{u}\gamma^\mu u + \bar{d}\gamma^\mu d + \text{other quarks})$$

where $u = (u_L, u_R)$, etc. – vector like U(1) symmetry. Same is true for the lepton number J_L^μ

- However, **only left-chiral components** couple to the SU(2) gauge field

- SU(2) gauge fields: three gauge components A_μ^a, $a = 1, 2, 3$

- The non-conservation of baryon and lepton numbers is given by

$$\partial_\mu J_B^\mu = \partial_\mu J_L^\mu = \frac{N_f g^2}{32\pi^2} \sum_{a=1}^{3} \mathcal{F}_{\mu\nu}^a \tilde{\mathcal{F}}_a^{\mu\nu}$$

where $\mathcal{F}_{\mu\nu}$ is the SU(2) field strength:

$$\mathcal{F}^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc} A^b_\mu A^c_\nu$$

$\tilde{\mathcal{F}}_{\mu\nu} = \frac{1}{4}\epsilon_{\mu\nu\alpha\beta}\mathcal{F}^{\alpha\beta}$ is the **dual field strength** ($\epsilon_{\mu\nu\alpha\beta}$ is the Levi-Civita tensor)

Problems: Recall that in the electrodynamics the electric and magnetic fields are defined from the field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ as $E_i = F_{0i}$ and $H_i = \frac{1}{2}\epsilon_{ijk}F^{jk}$, where $i, j, k = 1, 2, 3$ are spatial indexes

- Show that the dual field strength tensor $\tilde{F}_{\mu\nu} = \frac{1}{4}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$ interchanges $\vec{E} \leftrightarrow \vec{H}$, i.e. $H_i = \tilde{F}_{0i}$ and $E_i = \frac{1}{2}\epsilon_{ijk}\tilde{F}^{jk}$
- Show that $F_{\mu\nu}F^{\mu\nu} = \tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} = \vec{E}^2 - \vec{H}^2$
- Show that $F_{\mu\nu}\tilde{F}^{\mu\nu} = 4\vec{E}\cdot\vec{H}$
- Show that $F_{\mu\nu}\tilde{F}^{\mu\nu}$ is full 4-divergence, i.e. $F_{\mu\nu}\tilde{F}^{\mu\nu} = \partial_\mu K^\mu$ where K^μ is some 4-vector

- The total baryon number non-conservation is given by

$$\Delta B = \int_{t_1}^{t_2} dt \frac{dB}{dt} = \frac{N_f g^2}{8\pi^2} \int_{t_1}^{t_2} dt \int dV \sum_{a=1}^{3} \vec{\mathcal{E}}_a \cdot \vec{\mathcal{H}}_a$$

$N_f = 3$ – number of generations of fermions

g – SU(2) coupling constant, $\frac{g^2}{4\pi} \sim \frac{1}{30}$

$\mathcal{E}_i^a \equiv \mathcal{F}_{0i}^a$; $\mathcal{H}_i^a \equiv \tilde{\mathcal{F}}_{0i}^a$

- Recall that in U(1) theory

$$\int dV \vec{E} \cdot \vec{B} = \frac{dN_{\text{CS}}}{dt}$$

where we defined the **Chern-Simons number**

$$N_{\text{CS}} = \frac{g^2}{96\pi^2} \int d^3x \vec{A} \cdot \vec{B}$$

a very similar formula holds for non-Abelian SU(2) fields

To generate non-zero baryon number in transitions from $t_1 \to t_2$, we need fluctuations of the SU(2) gauge that change N_{CS}:

$$B(t_2) - B(t_1) = N_f \Big[N_{\text{CS}}(t_2) - N_{\text{CS}}(t_1)\Big]$$

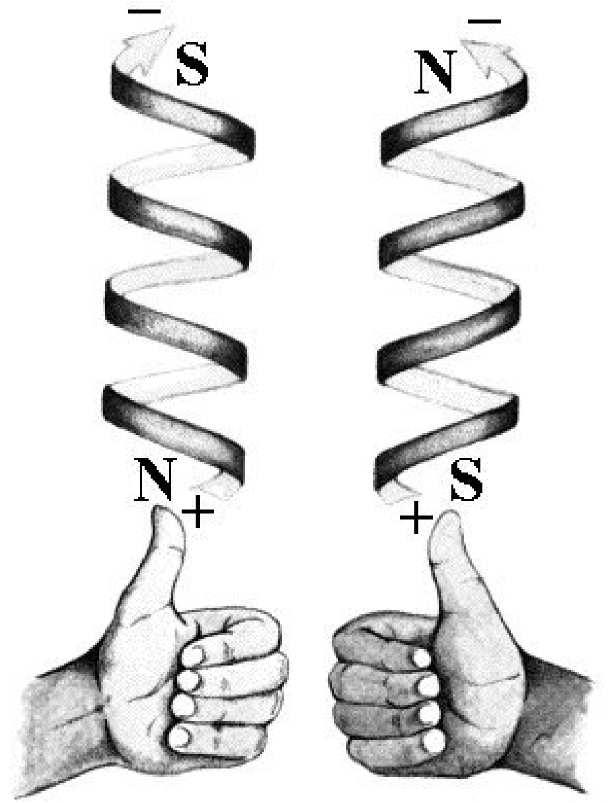
Immediate questions:

- If baryon number is not conserved in the Standard Model – why proton is stable?
- How possible are the configurations with non-zero $\vec{\mathcal{E}}_a \cdot \vec{\mathcal{H}}_a$?? How often do they occur at $T = 0$ and at high temperatures?
- $\Delta B \propto g^2 \int \vec{\mathcal{E}} \cdot \vec{\mathcal{H}} \Longrightarrow \vec{\mathcal{E}} \cdot \vec{\mathcal{H}} \propto \frac{1}{g^2}$ $\Rightarrow$ energy density of the SU(2) gauge field is $\propto \vec{\mathcal{E}}^2 + \vec{\mathcal{H}}^2 \propto \frac{1}{g^2}$. What do they mean?

Chern–Simons number

- Example: configuration with $\vec{A}(\vec{x}) = A_0\Big(\sin(kz), \cos(kz), 0\Big)$ has the magnetic field $\vec{B} = \mathrm{curl}\,\vec{A} = \vec{B}(\vec{x}) = k\vec{A}(\vec{x})$
- Magnetic energy density

$$\rho_B = \lim_{V\to\infty}\frac{1}{V}\int dV\,\frac{\vec{B}^2}{2} = \frac{1}{2}k^2A_0^2$$



- Its Chern-Simons number density:

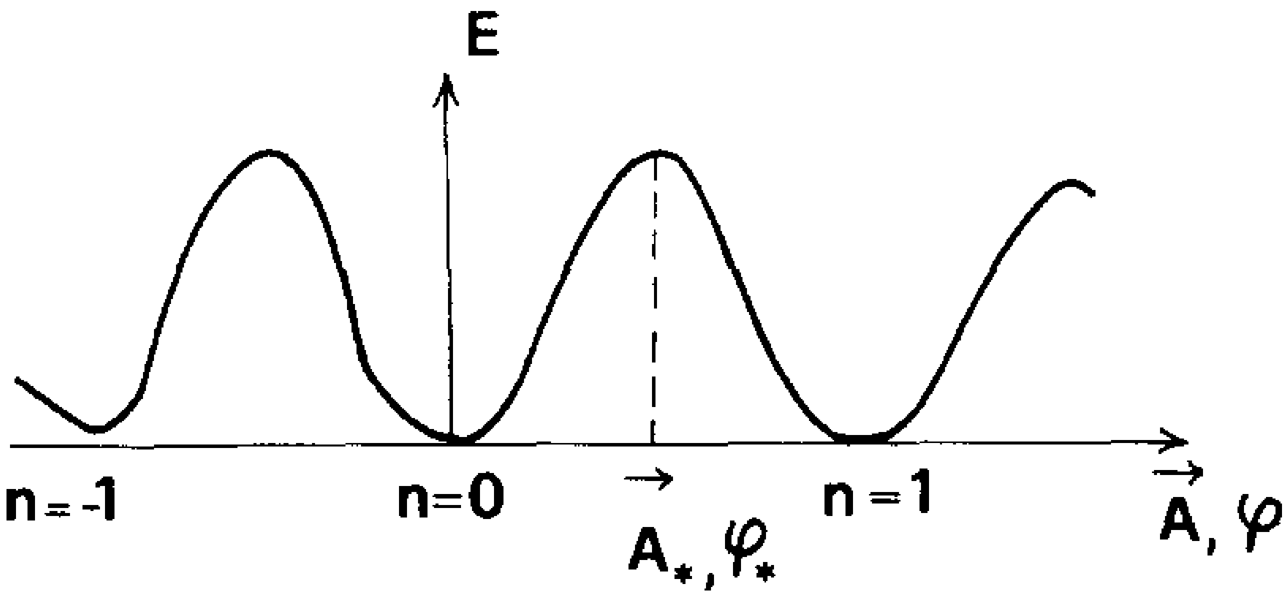
$$n_{\mathrm{CS}} = \lim_{V\to\infty}\frac{1}{V}\int dV\,\vec{A}\cdot\vec{B} = kA_0^2$$

- $N_{\mathrm{CS}} \neq 0$ means that the field is “helical”
- Notice that we can send $k \to 0$ and $A_0 \to \infty$ so that

$$n_{\mathrm{CS}} = \mathrm{const} \quad \text{but} \quad \rho_B \to 0$$

$\Rightarrow$ Configurations with $n_{\text{CS}} \neq 0$ have same energy as vacuum ($\vec{B} = 0$). Arbitrary configuration of gauge fields has higher energy

- The same is true for the SU(2) + Higgs field system:



- The height of this barrier is

$$E_{\text{barrier}} \approx \frac{2M_W}{\alpha_W} \sim 10\,\text{TeV}$$

Transitions with $\Delta N_{\mathrm{cs}} \neq 0$

- At zero energy to change N_{CS} one needs to tunnel through the barrier. The probability is given by

$$P_{\text{tunnel}} \sim e^{-\frac{4\pi}{\alpha_W}} \sim 10^{-160}$$

– therefore proton is stable

- At finite temperatures the rate of transition becomes unsuppressed:

$$\Gamma_{\text{sph}}(T) = \begin{cases} (\alpha_W T)^4 \alpha_W \log(1/\alpha_W), & T \gtrsim E_{\text{barrier}} \\ (\alpha_W T)^4 \left(\frac{E_{\text{barrier}}}{T}\right)^7 \exp\left(-\frac{E_{\text{barrier}}}{T}\right), & T \lesssim E_{\text{barrier}} \\ \exp\left(-\frac{4\pi}{\alpha_W}\right), & T = 0 \end{cases}$$

- Each fluctuation of SU(2) field with $\Delta N_{\text{CS}} = 1$ creates 9 quarks $+$ 3 leptons violating both baryon and lepton number but leaving $B-L$ conserved

Sakharov condition-II

- C- and CP-symmetries change **all** charges including baryon number:
  $C\,|p\rangle = |\bar{p}\rangle$, $C\,|n\rangle = |\bar{n}\rangle$, $C\,|e^-\rangle = |e^+\rangle$, etc.

- If these symmetries were conserved in the early Universe this would mean that for any process, changing baryon number, there is another process, restoring baryon number. Namely, if

$$X_1 + X_2 + \cdots \to Y_1 + Y_2 + \dots$$

change baryon number by $+1$, then there is a process:

$$\bar{X}_1 + \bar{X}_2 + \cdots \to \bar{Y}_1 + \bar{Y}_2 + \dots$$

in which baryon number changes by -1 and **their probabilities are the same**.

- All CP-non-conservation effects in the SM are in the quark sector
- These are complex phases of Cabbibo-Kobayashi-Maskawa mass matrix
- In analogy with neutrino mass matrix, one needs at least 3 flavours to have a possibility for the presence of complex phase that cannot be removed by field redefinition (CP-violation)
- There are two types of quark matrices: M_u for “up-quarks” (u,c,t) and M_d for “down-quarks” (d,s,b). Up-sector can be made diagonal, and the M_d is non-diagonal in flavour space.
- Notice, that some elements of these matrices do not play role in physical processes and can be reabsorbed in the redefinition of fields

CP non-conservation in the SM at $T \neq 0$

- The lowest order in mass CP-non-invariant expression that is invariant under all possible quark fields redefinitions is given by

$$J_{\rm CP} = \mathrm{Im}\,\mathrm{Tr}\Big(M_u^4 M_d^4 M_u^2 M_d^2\Big) \propto m_t^4 m_b^4 m_c^2 m_s^2 \sin\delta_{CP} \sim 10^4\ \text{GeV}^{12}$$

  where $\delta_{\rm CP}$ is the CP-violating phase that can be measured from kaon decays

- Notice that $J_{\rm CP}/T_{\text{sph}}^{12} \sim 10^{-20}$, where T_{sph} is a temperature of sphaleron freeze-out – a number much smaller than the baryon asymmetry (that we expect to be of the order 10^{-10})

Deviation from thermal equilibrium

- In thermal equilibrium any quantity is defined in a unique way as a function of **temperature** and possibly a number of some **conserved charges** Q or corresponding **chemical potentials**

- To any equilibrium process (changing C, CP, B, or any other quantity) there is a reverse process, changing any charge in the opposite direction. As a result for example, the total baryon charge $\langle B \rangle$ will approach its equilibrium value $B_{\rm eq}(T, Q, \dots)$ – a unique function of T and values of other conserved charges.

- Notice, that if a model does not have any conserved charges $Q \neq 0$, than in equilibrium baryon number (also, lepton number or any other quantum number **will be equal to zero**:

  Density matrix $\hat{\varrho} = e^{-\hat{H}/T}$ is CPT invariant (CPT theorem), while any charge Q changes sign under CPT. Therefore **in equilibrium**

$$\langle Q \rangle = \mathrm{Tr}\Big(\hat{Q}\hat{\varrho}\Big) = \mathrm{Tr}\Big(\hat{Q}^{(\mathrm{CPT})}\hat{\varrho}\Big) = \mathrm{Tr}\Big(-\hat{Q}\hat{\varrho}\Big) \Longrightarrow \langle Q \rangle = 0$$

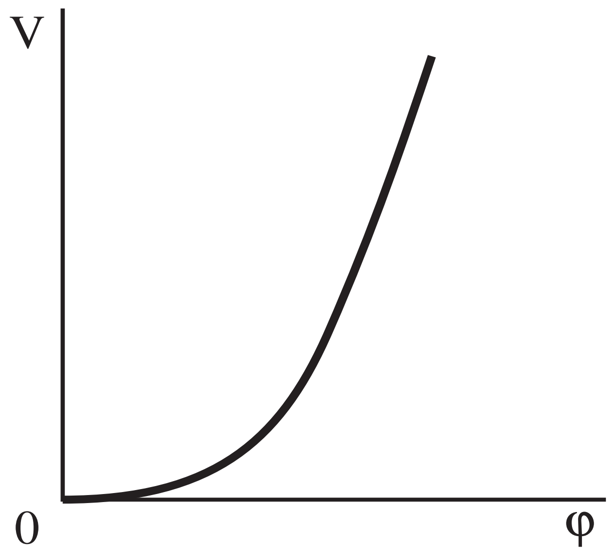
Problems about thermal equilibrium

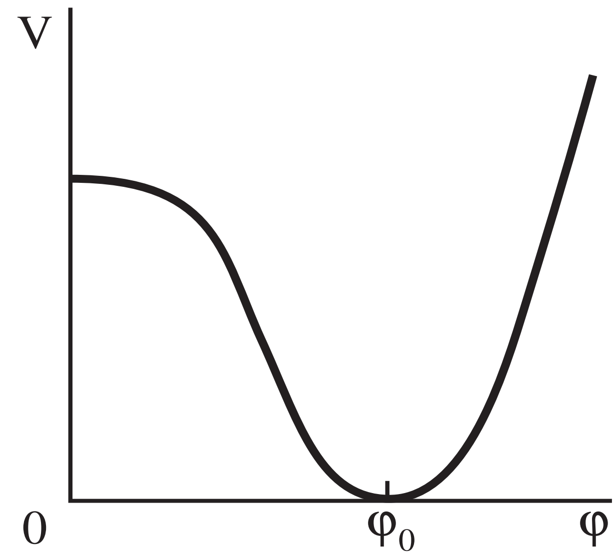
1. Estimate within the Fermi theory at what temperatures weak interactions enter thermal equilibrium? At what temperatures they go out of thermal equilibrium again?
2. At high temperatures ($T \gg 100$ GeV) one can speak about unbroken $SU(2) \times U(1)$ electroweak symmetry. The interaction is characterized by "weak oupling constant" $\alpha_W = \frac{g_W^2}{4\pi} \approx \frac{1}{30}$. Estimate, at what temperatures typical electroweak reactions are in thermal equilibrium? (Hint: use the analogy with the electromagnetic interactions)
3. Below temperatures $T \ll 100$ GeV electroweak symmetry is broken and one can speak about electromagnetic interactions. Estimate at what temperatures electromagnetic processes enter thermal equilibrium. At what temperatures they go out of thermal equilibrium again?
4. At temperatures $T \ll 100$ GeV interactions of sterile neutrino with other leptons can be described by the analog of Fermi theory (with the "sterile Fermi constant" $G'_F = \theta * G_F$, where $\theta \sim 10^{-5}$). At what temperatures such sterile neutrinos enter thermal equilibrium and go out of thermal equilibrium.

- We saw that all the SM processes containing quarks (particle, carrying baryon number) are in thermal equilibrium in the early Universe down to temperatures ~ 1 MeV when proton and neutron freeze-out. Therefore, they cannot be responsible for generation of baryon asymmetry of the Universe?

- What are physical processes can violate thermal equilibrium conditions?

PHASE TRANSITIONS IN THE EARLY UNIVERSE





Massive scalar

$$V(\phi) = \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$$

Spontaneous symmetry breaking

$$V_{\text{SSB}}(\phi) = -\frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + V_0 \qquad (12)$$

Minimum at $\boxed{\phi_0^2 = \frac{m^2}{\lambda}}$

1. Expand the potential (12) around the true minimum ϕ_0 and find the mass of the particle. Notice that it is not equal to m^2!

2. Generalize the potential (12) for the case of U(1) charged scalar field with the charge q (i.e. ϕ is the complex field and gauge transformation acts as $\phi \to e^{iq\xi}\phi$)

3. Introduce the gauge field A_μ and write the kinetic term for the charged scalar field ϕ (changing ordinary derivative ∂_μ to covariant derivative $D_\mu = \partial_\mu + qA_\mu$). Show that at the minimum of the potential $V_{\rm SBB}$ the gauge field becomes massive. Find the mass of the gauge field

4. Introduce the charged fermions, coupled to the field ϕ via **Yukawa interaction** $f\bar{\psi}\phi\psi$. Show that when ϕ is at the minimum of the potential (12) the Yukawa term becomes the Dirac mass term for the fermion.

Symmetry breaking at finite temperatures

- What happens with the system at finite temperatures (e.g. when $T^4 \sim V_0$)?

- Consider **free energy per unit volume** of the scalar field in the homogeneous and isotropic Universe:

$$\boxed{V_{\mathsf{EFF}}(\phi, T) \equiv \text{Free energy}/\text{Volume}}$$

- At low temperatures $V_{\mathsf{EFF}}(\phi, T) \approx V_{\mathrm{SBB}}(\phi)$

- The minimum of effective potential $\langle\phi\rangle_T$ is determined from the usual condition of minimum $\frac{\partial V_{\mathsf{EFF}}(\phi,T)}{\partial\phi} = 0$ while $\frac{\partial^2 V_{\mathsf{EFF}}(\phi,T)}{\partial\phi^2} > 0$

- What is the form of $V_{\text{EFF}}(\phi, T)$? Consider the situation when the temperature is high $T \gg \langle\phi\rangle_T$. Qualitatively in this situation all the particles have masses $m(\phi) \propto \langle\phi\rangle_T$ and $m(\phi) \ll T$. In this limit one would expect that the change in free energy density (is given by)

$$V_{\text{EFF}}(\phi, T) \approx V_{\text{SBB}}(\phi) + T^4 \sum_{\text{all massive particles}} c_i \frac{m_i^2(\phi)}{T^2}$$

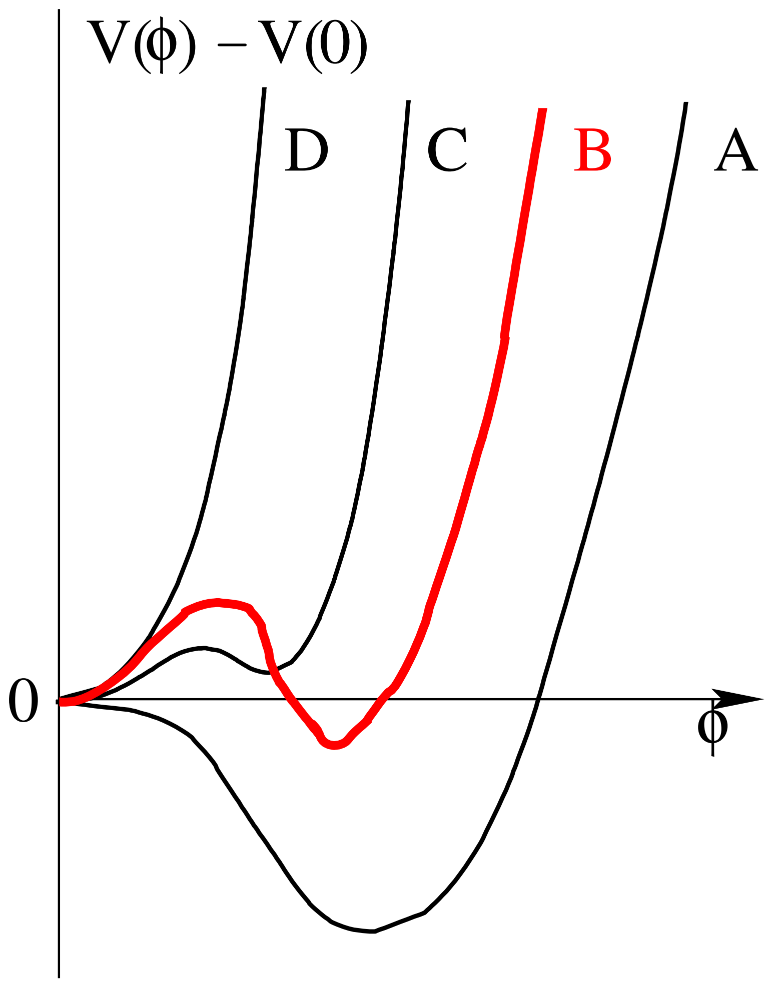
- Then we have

$$V_{\text{EFF}}(\phi, T) \approx (-\lambda v^2 + \alpha T^2)\phi^2 + \lambda\phi^4$$

where

$\alpha = \dfrac{1}{12v^2}(6M_W^2 + 3M_Z^2 + 6m_t^2)$ – heaviest particles to which Higgs couples

- If $T > \sqrt{\frac{\lambda}{\alpha}}v$ then $\phi = 0$ is the minimum of the system



Proper treatment of quantum corrections gives

$$V_{\text{EFF}} = \frac{\alpha}{24}(T^2 - T_{c2}^2)\phi^2 - \gamma T\phi^3 + \lambda\phi^4$$

D: single minimum at $\phi = 0$
symmetric phase

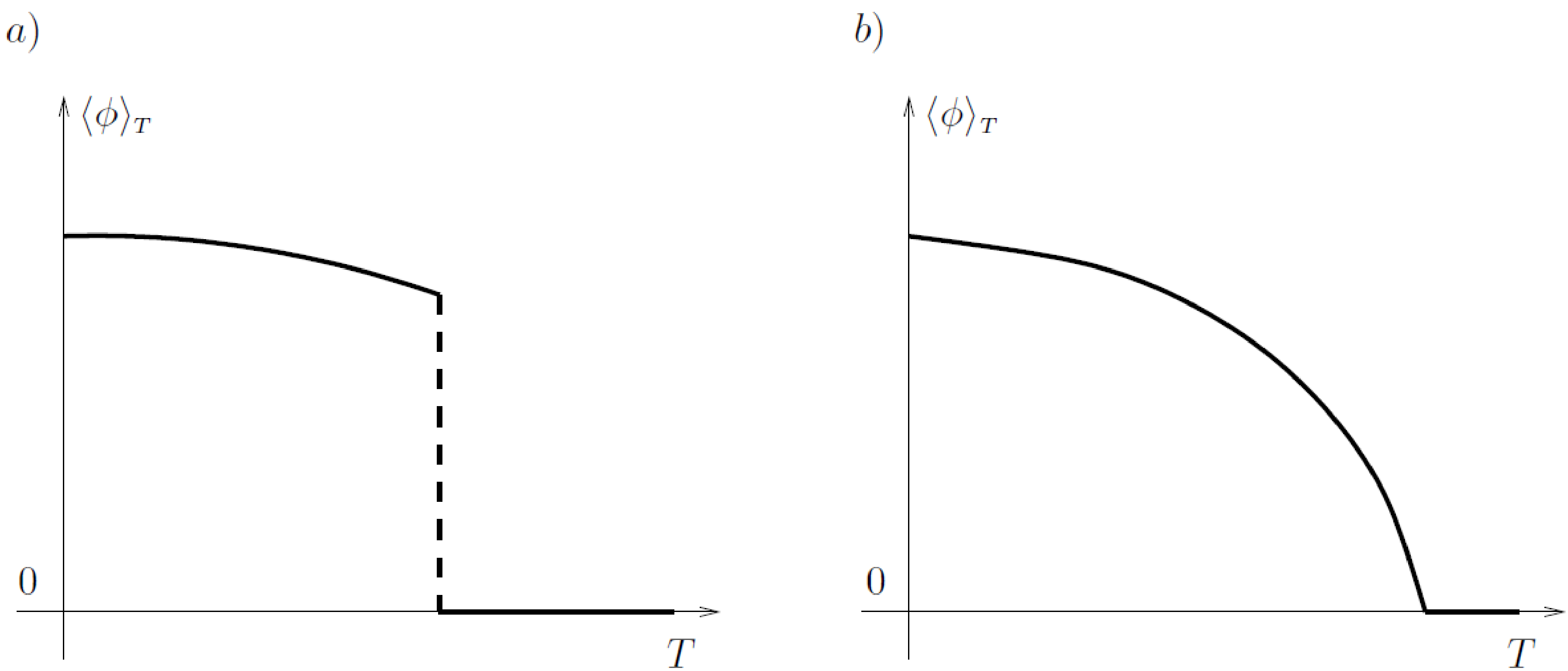
C: second (local) minimum appears at $\phi \neq 0$. The distance between two minima always finite!

B: Minimum at $\phi \neq 0$ is a true minimum separate from the metastable vacuum at $\phi = 0$ by potential barrier

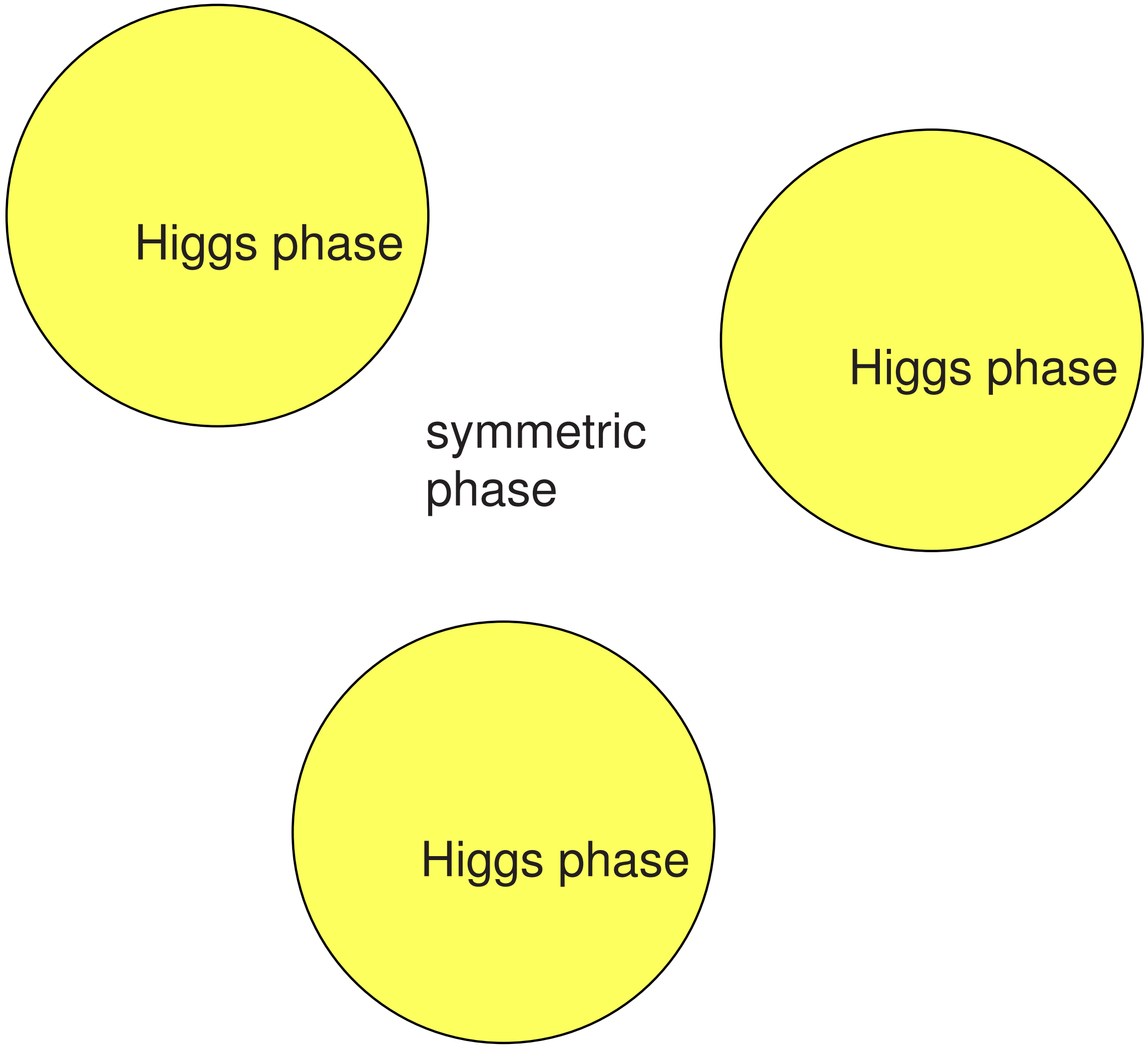
A: $T = 0$, $\phi_{min} = \phi_0$

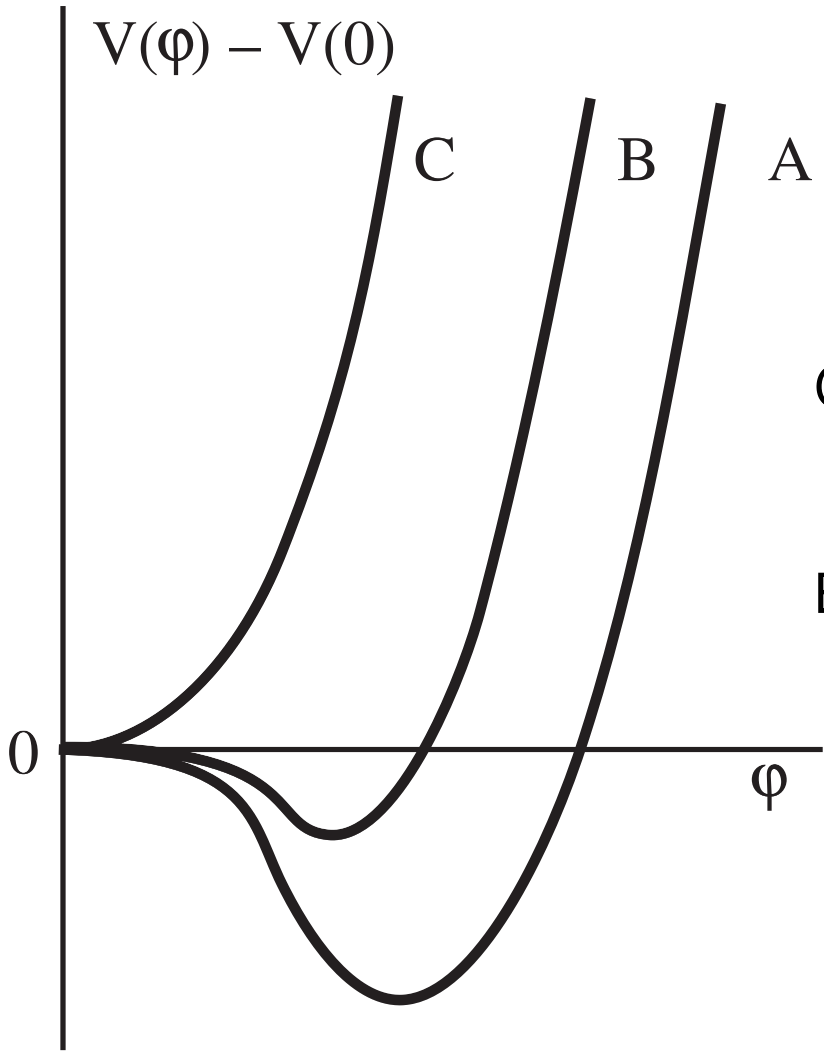
Two main types of phase transitions:

- **I order** – Discontinuity of $\frac{\partial F}{\partial T} \propto \langle\phi\rangle_T$ (**left**).
- **II order** – No discontinuity of $\langle\phi\rangle_T$ (**right**).



First-order phase transition





$$V_{\text{EFF}} = -\frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + \frac{\lambda T^2}{2}\phi^2$$

$$T_c = \frac{m}{\sqrt{\lambda}}$$

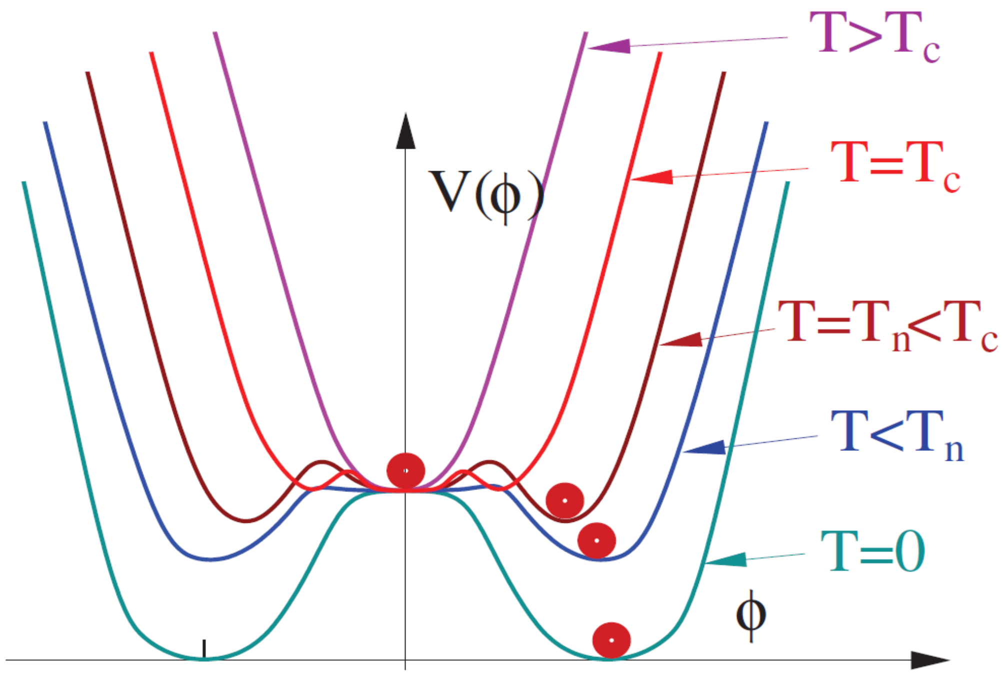
C: $T > T_c$ single minimum at $\phi = 0$
symmetric phase

B: At $T \leq T_c$ a minimum at $\phi \neq 0$ appears, $\phi = 0$ becomes maximum. $\langle\phi\rangle_T = \sqrt{\frac{T_c^2 - T^2}{\lambda}}$. Notice that $\langle\phi\rangle_T \propto \sqrt{T_c - T}$ – starts from 0 at $T = T_c$ (unlike the 1st order P.T.)

A: $T = 0$, $\phi_{min} = \phi_0$

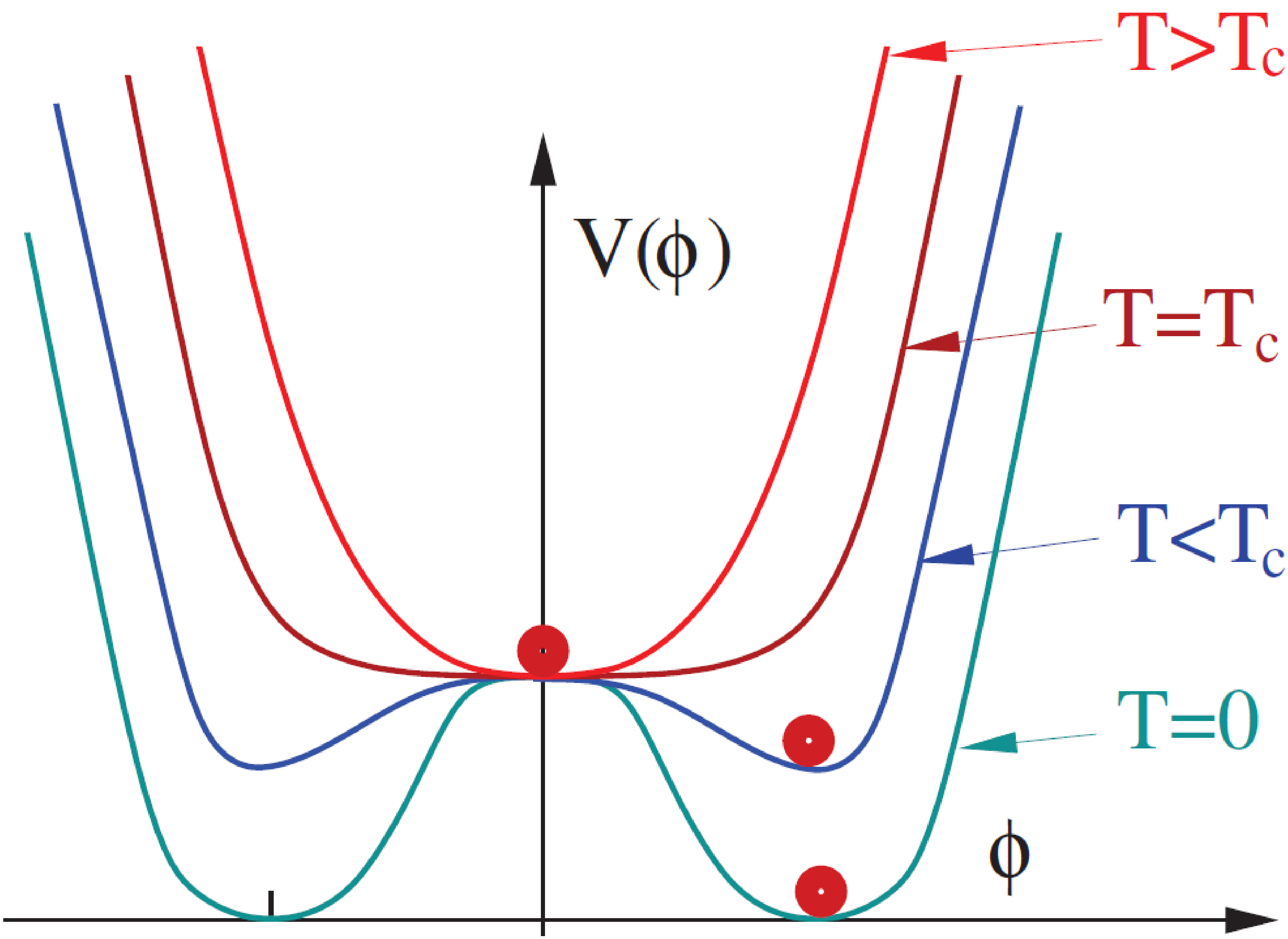
In the presence of the temperature, the potential for the field ϕ can change:

$$V_{\mathsf{EFF}}(\phi, T) = \lambda \left[\frac{\phi^4}{4} + \frac{\phi^2}{2} \left(\frac{T^2}{4} - v^2 \right) \right]$$



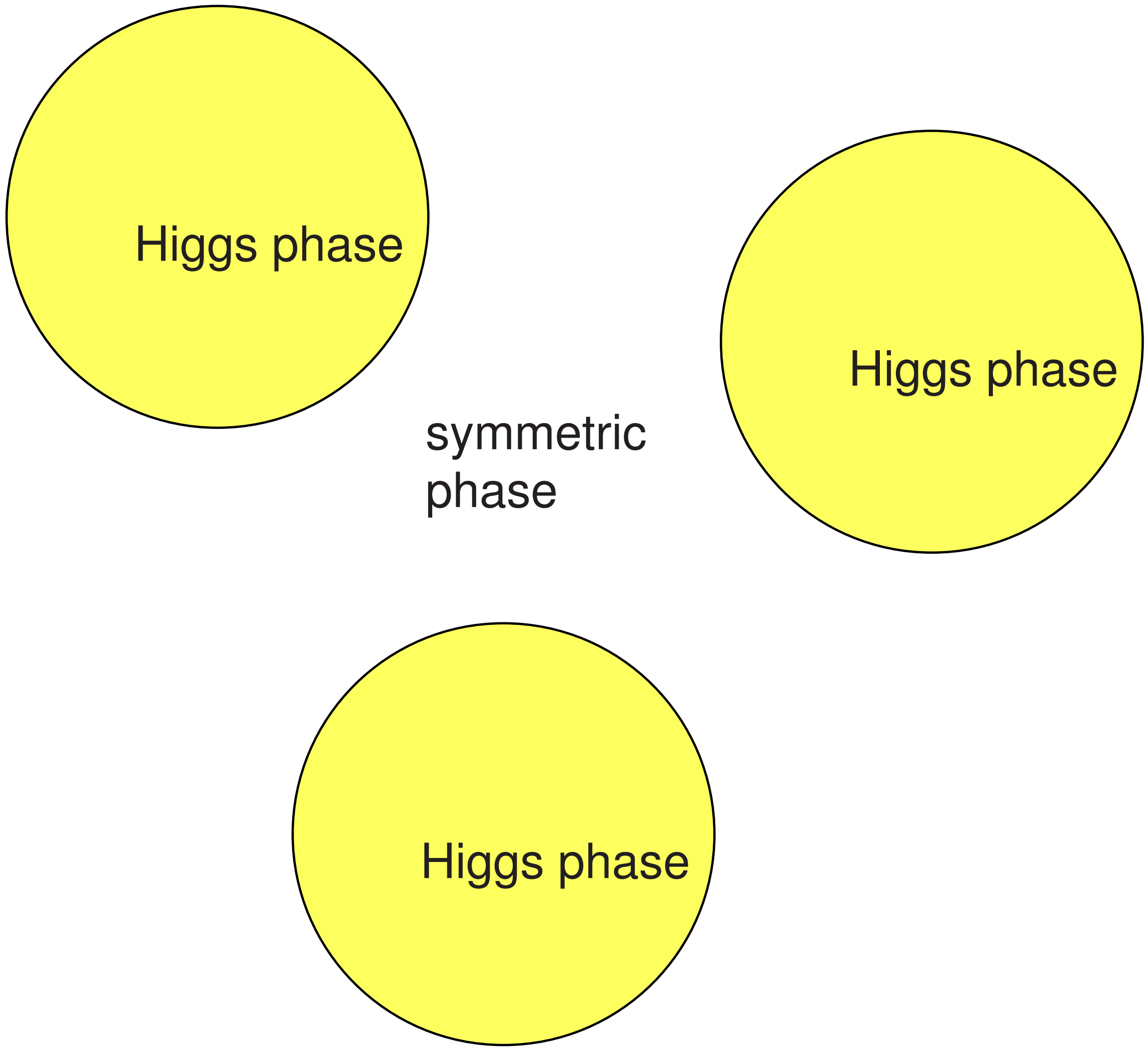
$T_c = 2v$

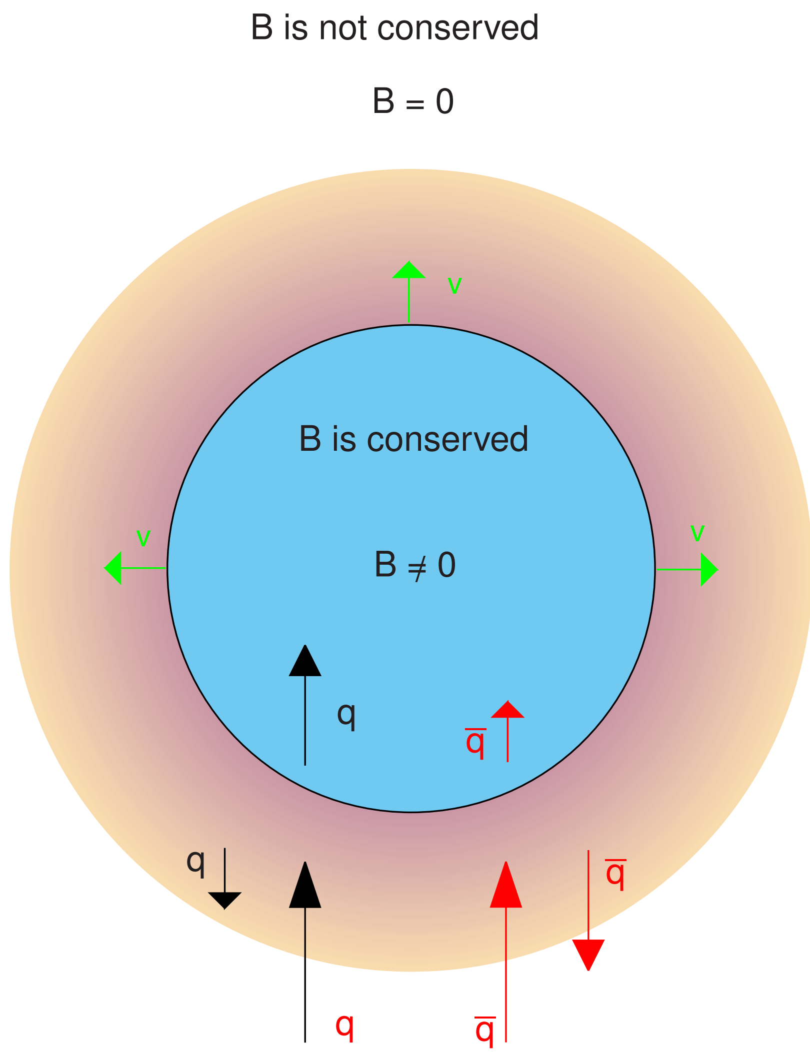
From `http://www.phys.uu.nl/~prokopec`



Back to Sakharov

First-order phase transition




B is not conserved
B = 0
v
v
v
B is conserved
B ≠ 0
q
$\bar{q}$
q
$\bar{q}$
q
$\bar{q}$

- In the SM all the conditions seems to be satisfied:
  - CP is violated
  - Baryon number may not be non-conserved: it can be created from lepton number by non-perturbative processes active at high temperature
  - There may be phase transitions (EW, QCD).
- However, experimental bounds on the SM parameters show that this does not happen!

- Sphaleron processes violate $B + L$ but do not affect $B - L$ charge.
- If at $T > T_{\text{sph}}$ you generate non-zero **lepton number** via some process (so that $B - L$ becomes $\neq 0$) ...
- ...then sphalerons will "**transform L into B**" (so that, for example, in the SM plasma one gets $B = \frac{28}{79}(B - L)$ and $L = -\frac{51}{79}(B - L)$)

Khlebnikov & Shaposhnikov, "*The Statistical Theory of Anomalous Fermion Number Nonconservation*" Nucl. Phys. B308 (1988) 885-912

This class of scenarios is called LEPTOGENESIS

Sterile neutrinos and leptogenesis

There exist three classes of leptogenesis scenario related to sterile neutrinos:

Thermal leptogenesis: Fukugita & Yanagida'86

Works for $M_N \sim 10^{12}$ GeV

Resonant leptogenesis: Pilaftsis, Underwood'04–'05

Works for $M_{N_1} \approx M_{N_2} \sim M_W$ and $|M_{N_1} - M_{N_2}| \ll M_{N_1,N_2}$

Leptogenesis via oscillations: Akhmedov, Smirnov & Rubakov'98

Asaka &Shaposhnikov'05

Works for $M_{N_1} \approx M_{N_2} \lesssim M_W$ and $|M_{N_1} - M_{N_2}| \ll M_{N_1,N_2}$

The main idea of thermal leptogenesis

- "Sufficiently heavy" sterile neutrinos can decay into left leptons + Higgs

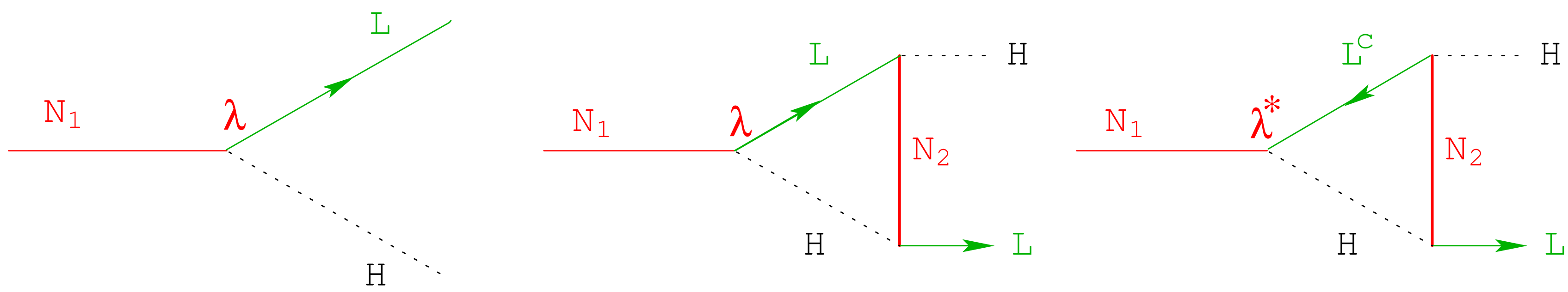


Fig. from **Strumia & Vissani**

- Due to their Majorana mass these decays break **lepton number** ($N \to L + H$ and $N \to \bar{L} + \bar{H}$)

- Decay rate $\Gamma_{tot} \propto |F|^2 M_N$

- Tree level decay of $N_1 \to L + H$ (the first graph):

$$\Gamma = \frac{|F|^2 M_1}{8\pi}$$

– complex phase does not contributes, so not satisfied **2nd Sakharov condition**

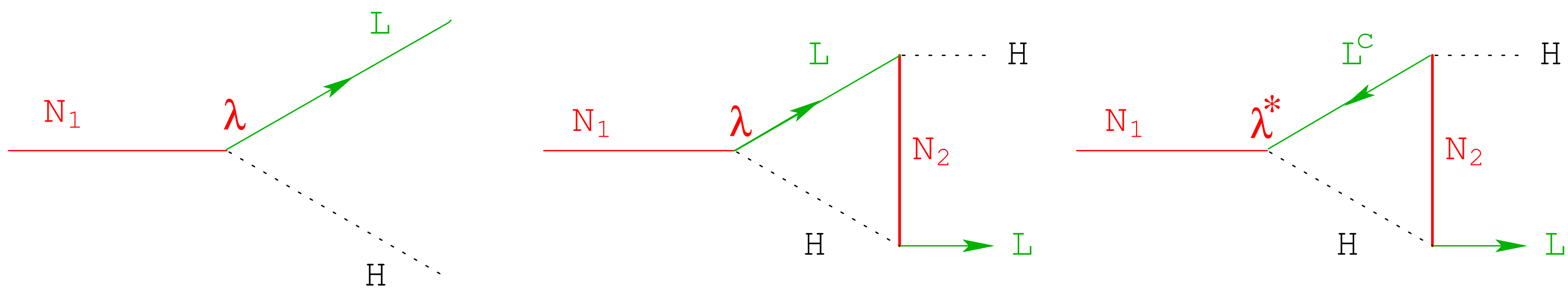
- Need to take into account loop effects (graphs 2 and 3)

- The resulting $\eta_B \propto 10^{-3}|F|^2 \Rightarrow |F|^2 \sim 10^{-7} \Rightarrow \boxed{M_N \sim 10^{12}\ \text{GeV}}$

- Such sterile neutrinos would add corrections to the Higgs mass of the order of $|F|^2 M_N^2 \sim 10^{14}\ \text{GeV}^2 \gg M_{\text{Higgs}}^2$ — **gauge hierarchy problem!**

- Can lighter sterile neutrinos provide leptogenesis?

- **Yes!** but still $\eta_B \propto |F|^2$ and one needs to compensate smaller Yukawas (the smaller is the mass, the smaller are the Yukawa couplings)



- If masses of two sterile neutrinos **approximately equal**, then in the last diagram the production of lepton number is enhanced by $\dfrac{M_1\Gamma_{tot}}{(M_1 - M_2)^2 + \Gamma_{tot}^2}$

- Leptogenesis possible for $M_N \sim M_W$!

Akhmedov, Smirnov, Rubakov'98

Asaka, Shaposhnikov '05

- As M_N decreases, the Majorana nature of particles plays lesser role. Can one get a leptogenesis for $M_N \ll T_{\text{sph}}$
- Recall, that sphalerons (SU(2) gauge configurations) convert **lepton number stored in left lepton doublets** into the baryon number.
- Can it be that **total lepton number** $= 0$ but is distributed between sectors:

$$\text{Lepton \# of left } \nu = -\text{Lepton \# of } N_I \neq 0$$

sterile neutrinos are Majorana particles, so for them role of lepton number is played by helicity

- Need at least **two sterile neutrinos** with $M_{N_1} \approx M_{N_2}$

Leptogenesis via oscillations

Akhmedov, Smirnov, Rubakov'98

1) Need to choose **at least two** sterile neutrinos that **do not thermalize** until T_{sph}

- At $T > m_t$ thermalization goes via Higgs exchange $N + t \leftrightarrow \nu + t$ or $H \leftrightarrow N + \nu, \ldots$
- $\Gamma_{therm} \sim \frac{9|F|^2 f_t^2}{64\pi^3}$ compares to $H(T)$ at $T_{eq} \sim 5M_N$.
- Therefore, if $M_N < M_W$ – particles not thermalized until sphalerons freeze-out

2) Sterile neutrinos are produced (e.g. via $H \to N + \nu$)
no lepton number is any sectors so far!

3) Sterile neutrinos **oscillate into each other** in the CP-violating way
recall, that we can have CP-phases in the Yukawa matrix of sterile neutrinos!

4) This generate some effective "lepton number" in sterile (and, therefore, in active) sectors

- The frequency oscillation between two neutrinos with masses $M_1 \approx M_2$ is given by

$$\omega_{\text{osc}} \sim \frac{M_1^2 - M_2^2}{E_N} \approx \frac{M_N \Delta M(T)}{T}$$

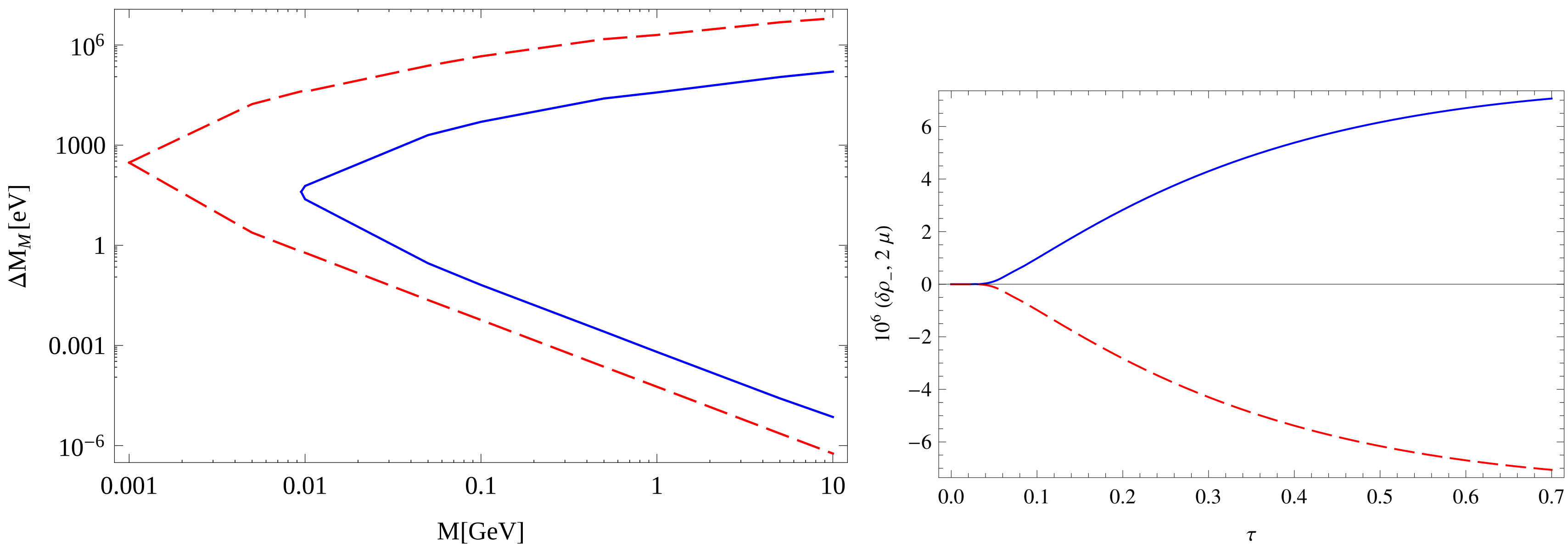
- If $\omega_{\text{osc}} \gg H(T_{\text{sph}})$ – many oscillations had occurred by the time of sphaleron freeze-out and lepton number is "washed out"

- If $\omega_{\text{osc}} \ll H(T_{\text{sph}})$ – essentially no oscillations had occurred and lepton number in the sterile sector did not have time to develop

- Optimal condition:

$$\omega_{\text{osc}} \sim \frac{T_{\text{sph}}^2}{M_*} \Longrightarrow \big(M_* M_N \Delta M(T)\big)^{1/3} \sim T_{\text{sph}}$$

Leptogenesis via oscillations



Canetti & Shaposhnikov [1006.0133]

- Mechanism works down to $M_N \sim 1$ MeV

- Roughly $\Delta M(T_{\text{sph}}) \sim m_{\text{atm}} \left(\frac{1 \text{ GeV}}{M_N}\right)$

- This leptogenesis do no have to stop at $T \sim T_{\text{sph}}$. Lepton asymmetry continues to be generated **below sphaleron freeze-out**

Baryon asymmetry in the Universe

- Sterile neutrinos with the masses from 1 MeV to 10^{12} GeV can be responsible for generation of baryon asymmetry of the Universe through leptogenesis

- Heavy particles ($M_N \sim 10^{12}$ GeV) would lead to the gauge hierarchy problem

- Almost degenerate particles with the masses from $M_N \sim M_W$ can produce baryon asymmetry through either **resonant leptogenesis** (Majorana nature of particles plays crucial role) or via **coherent CP-violating oscillations** – total lepton number is non-zero but the active neutrinos acquire an effective lepton number

- The latter mechanism allows for lepton asymmetry to be generated below the sphaleron freeze-out temperature. Therefore, it is possible that $\eta_L \gg \eta_B$ in such models