Particle Physics of the early Universe

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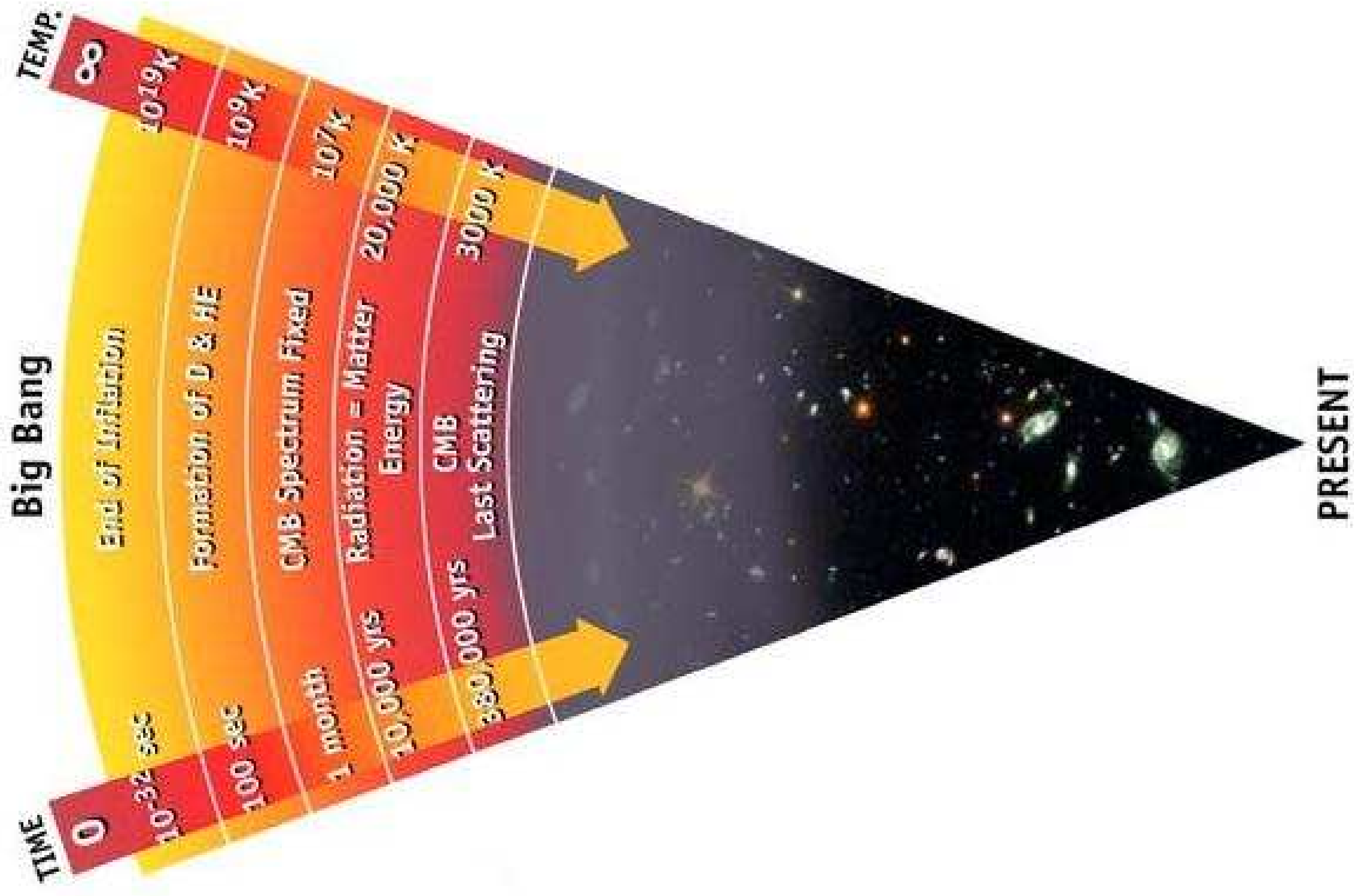
Early Universe

Thermal history of the Universe

Today you all got used to pictures like this



HOW DID WE LEARN ALL THAT?

- **Einstein** applys GR to the whole Universe **assuming** spatial homogeneity and isotropy (for isotropy there were observational evidence, for homogeneity — it was a bold extrapolation, due to Hubble's observations of fainter and fainter "nebulae")

1917 – early 1920s

- The metric is given by

$$ds^2 = -dt^2 + \underbrace{R^2(d\chi^2 + \sin^2\chi d\theta^2 + \sin^2\chi\sin^2\theta d\phi^2)}_{\text{3-sphere}}$$

– static cylinder

- Closed Universe – finite total volume $V = 2\pi^2 R^3$

Cosmological model of Einstein

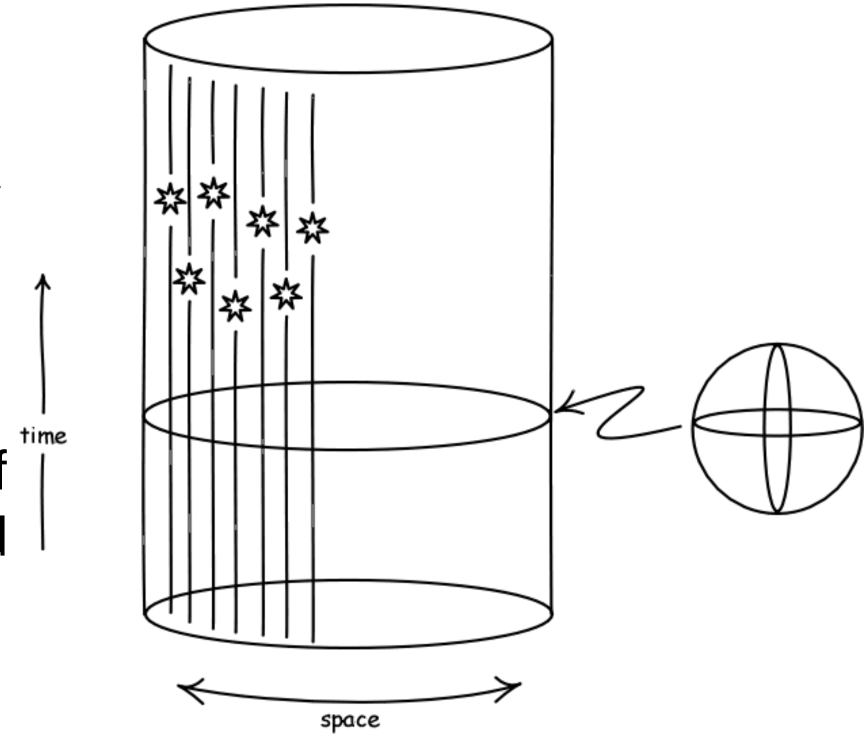
- Plug this metric into the Einstein's equation:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \boldsymbol{\Lambda} g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$T_{\mu\nu} = \mathrm{diag}(\rho, -p, -p, -p)$$

- The solution exists if cosmological constant and matter are related as

$$\Lambda = \frac{1}{R^2}, \quad \rho = \frac{2}{8\pi G R^2}$$



- Total mass of the Universe $M = \rho \cdot 3\pi^2 R^3 = \frac{\pi R}{2G}$

- Everything is a function of density that can be measured experimentally $\Rightarrow$ **full solution of the Universe constructed?**

Cosmological model continued

- **de Sitter** (1917) finds a different solution

$$ds^2 = -R^2 \cos^2 \chi dt^2 + \underbrace{R^2(d\chi^2 + \sin^2 \chi d\theta^2 + \sin^2 \chi \sin^2 \theta d\phi^2)}_{\text{3-sphere}}$$

- To satisfy GR equations this requires

$$\Lambda = \frac{3}{R^2}, \quad \rho = 0$$

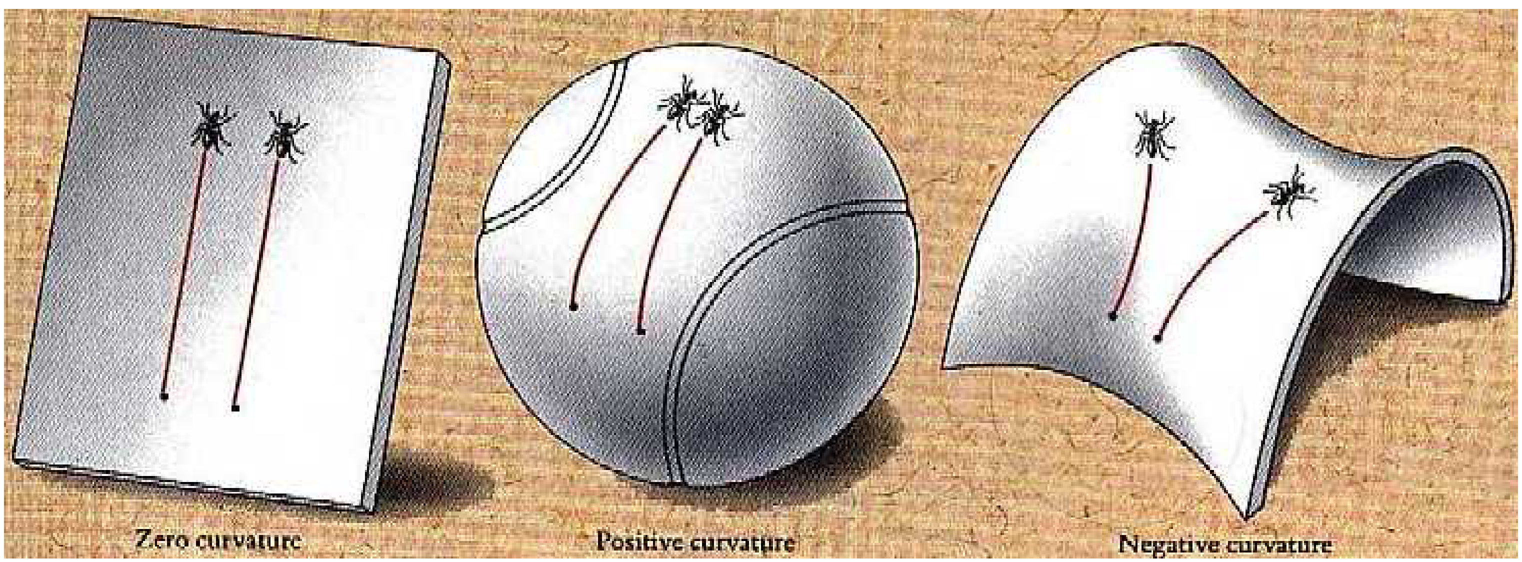
– curved Universe without matter??

1922

- Friedmann write the general ansatz for **homogeneous** and **isotropic** metric

$$ds^2 = -dt^2 + a^2(t)\left(\frac{d\chi^2}{1-\kappa\chi^2} + \chi^2 d\theta^2 + \chi^2 \sin^2\theta d\phi^2\right), \qquad \kappa = -1, 0, 1$$

- Three homogeneous and isotropic spaces (κ – sign of curvature)



Cosmological model continued

- Plug this metric into the Einstein's equation, using the general form of the stress-energy tensor being

$$T_{\mu\nu} = \mathrm{diag}(\rho, -p, -p, -p)$$

- The Eistein's equations relate "matter" (some functions $\rho(t)$ and $p(t)$) with the dynamics of the **scale factor** – Friedmann equation:

$$\frac{\dot{a}^2(t)}{a^2(t)} \equiv H^2(t) = \frac{8\pi G}{3}\rho - \frac{\kappa}{a^2}$$

1922-1924

- Second Friedmann equation:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

Cosmological model continued

- Energy conservation:

$$\frac{\partial\rho}{\partial t} = -3H(\rho+p) = -3\frac{\dot{a}}{a}(\rho+p)$$

- Lemaître rediscovers these equations

- Main predictions: the Universe is expanding. Static Universe would require very specific equations of state ($\rho = -\kappa\frac{3}{8\pi G}R^2_{\text{static}}$ and $\rho = -3p$). Such a solution will be nevertheless **unstable**

Problems 1a-1c

Cosmology in a couple of words

- **Matter-dominated Universe**: $p = 0$ and $\frac{\partial\rho}{\partial t} = -3H\rho$ or $\rho a^3 = \mathrm{const}$ and $a \propto t^{2/3}$

- **Radiation-dominated Universe**: $p = \frac{1}{3}\rho$ and $\frac{\partial\rho}{\partial t} = -4H\rho$ or $\rho a^4 = \mathrm{const}$ and $a \propto t^{1/2}$

- Temperature $T \propto a^{-1}$. In radiation-dominated epoch $\rho = \frac{\pi^2}{30} g_{\mathsf{EFF}} T^4$

- **Einstein's Λ-term:** $\rho(t) = -p(t) = \mathrm{const}$, $a = e^{\sqrt{\frac{\Lambda}{3}}t}$

- Hubble equation — interplay between **kinetic energy** $E_k = \frac{\dot{a}^2}{2}$ and **potential energy** $E_p = -\frac{GM}{a(t)}$:

$$\frac{\dot{a}^2}{2} - \frac{G\frac{4\pi}{3}\rho(t)a^3(t)}{a(t)} = -\frac{\kappa}{2}$$

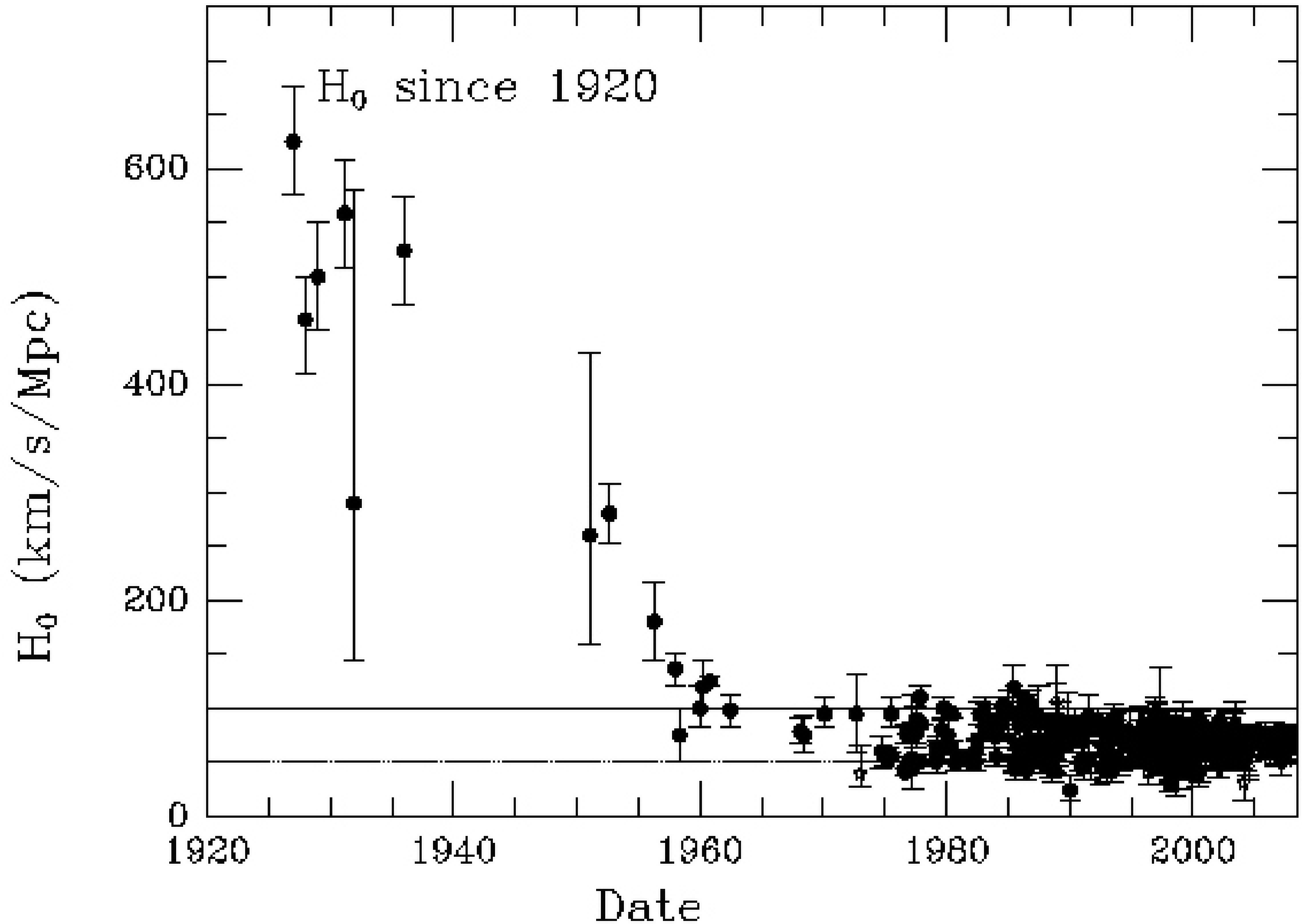
- Slipher discovers redshifts of the spectral lines in the nearby galaxies . De Sitter speculates for the first time that this can be due to cosmological expansion in his model 1912-1913

- **Hubble** discovers that “spiral nebulae” are far from us (M31, M33) 1925

- Hubble estimates the distance to the nearby galaxies and establishes redshift-distance relation 1926

$$cz = H_0 r$$

Hubble constant history

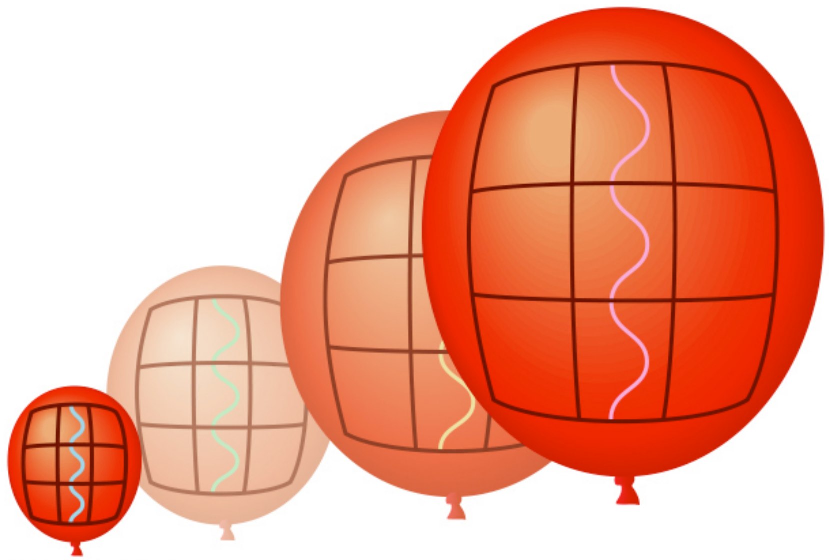


https://www.cfa.harvard.edu/~dfabricant/huchra/hubble

Expansion of the Universe – **the first pillar of cosmology**

Universe stretches:

$$\color{red}{1+z} \equiv \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} = \frac{a(t_{\text{obs}})}{a(t_{\text{emit}})}$$



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Doppler effect:

a galaxy is receding

$$\color{red}{1+z} \equiv \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} = \sqrt{\frac{1+v/c}{1-v/c}}$$

where Hubble velocity
$v = H_0 \times$ distance

The initial state of the Universe remained a problem

- If Universe is filled with cosmological constant – its energy density does not change
- If Universe is filled with anything with non-negative pressure: the density decreases as the Universe expands

In the past the Universe was becoming denser and denser, $\rho \propto \frac{1}{t^2}$,
$\Longrightarrow$ ultradense cold state of the initial Universe?

High density baryonic matter — a Universe-size **neutron star**? Neutrons cannot decay anymore ($n \to p + e + \bar{\nu}_e$) as there are no available Fermi levels for fermions. The state is stable and remains such until cosmological singularity ($\rho \propto 1/t^n$)

Problems 1c,2a,5a

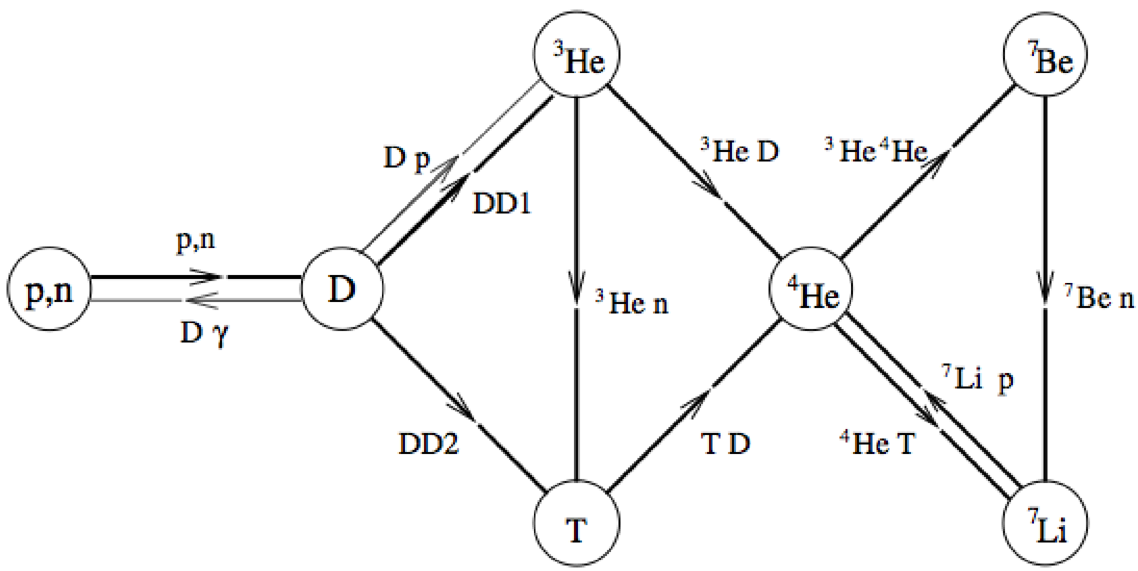
The origin of elements (Hydrogen, Helium, metals) remained a challenging problem

1920s-1930s

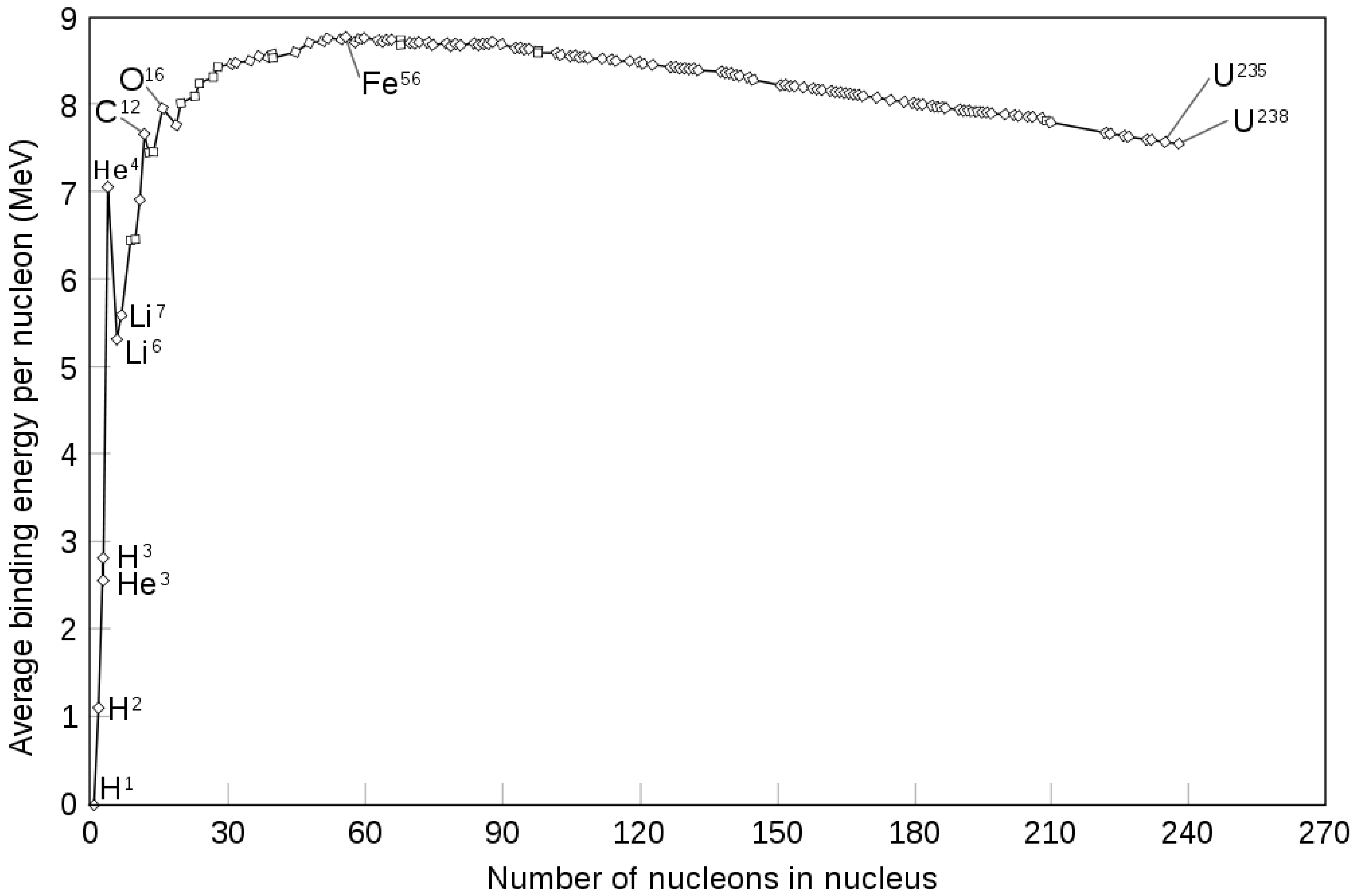
Ultradense ($\rho_n \sim 1\mathrm{g/cm}^3$) **neutron star** would mean that **no hydrogen is left** (as soon as density has dropped to allow neutron decay $n \to p + e + \bar{\nu}_e$, each proton is bombarded by many neutrons so that $p + n \to d + \gamma$, $d + n \to t + \gamma$)

See e.g. review by Zel'dovich, Section 13

Zel'dovich in Wikipedia or here



Binding energy



Nucleosynthesis

- If (as people thought) the density of plasma, needed for nuclear reactions to take place was $\rho \sim 10^7\,\mathrm{g/cm^3}$ $\Rightarrow$very rapid expansion of the Universe (age $\sim 10^{-2}\,\mathrm{sec}$). Not enough to establish thermal equilibrium?

Paper by Gamow (1946

- **Gamow suggested** nuclear reactions take place because the Universe is **hot** rather than "ultradense"

Paper by Gamow (1948

- Consider the plasma **temperature** $T_d \sim 10^9$ K (~ 100 keV) (order of the binding energy of nuclei)

- He computed the total energy density of radiation as

$$\rho_{\text{rad}} = \sigma_{\text{SB}} T^4 = 8.4\,\mathrm{g/cm^3} \left(\frac{T}{10^9\,\mathrm{K}}\right)^4$$

- Gamow then assumed that the energy density of the Universe is dominated by radiation and estimated its age as

$$\rho_{\text{rad}} = \frac{3}{32\pi G_N}\frac{1}{t^2} \quad \text{or} \quad t\,[\text{sec}] \sim \frac{1}{(T\,[\text{MeV}])^2}$$

- The age of the Universe at $T_d = 10^9$ K is equal to $t_d \sim 10^2$ sec
- Gamow estimates the density of matter by demanding that the time between collisions is equal to the **age of the Universe** t_d:

$$\underbrace{\left(v\, n_b\, \sigma_{p+n\to d+\gamma}\right)^{-1}}_{\text{time between } p\text{-}n \text{collisions}} \sim t_{\rm d}$$

- Gamow knew that the cross-section $\sigma \sim 10^{-29}\,\text{cm}^2$, and computed thermal velocity $v \sim \sqrt{\frac{T}{m}}$ ⇒one gets $n_b^{(i)} \sim 10^{18}\,\text{cm}^{-3}$ and therefore $\rho_b^{(i)} \sim 10^{-6}\,\text{g/cm}^3 \ll 8.4\,\text{g/cm}^3$ ⇒**the Universe was radiation dominated**!

- Cross-section is the effective area that each incoming particle ”sees” – the probability of some scattering event.

- At this temperature, $T_d \sim 10^9$ K the number of photons is given by

$$n_\gamma = \frac{2\zeta(3)}{\pi^2} T_d^3 \approx 10^{28}\,\mathrm{cm}^{-3} \left(\frac{T}{10^9\,\mathrm{K}}\right)^3 \quad (1)$$

- Prediction of the Gamow’s theory:

Baryon-to-photon ratio $\eta_B \equiv \frac{n_b}{n_\gamma} \sim 10^{-10}$

Relic radiation

- Second prediction: the Universe **today** should have a bath of thermal photons, left from that time
- Baryon-to-photon ratio η_B is constant with time. Indeed, $na^3 \approx \mathrm{const}$ and $Ta \approx \mathrm{const}$ (indeed, for radiation $\rho \propto T^4$ and $\rho \propto a^{-4}$)
- What is the temperature of radiation bath today? All matter today was considered to be baryonic. So $H_0^2 = \frac{8\pi G}{3}\rho_b = \frac{8\pi G}{3} m_{\text{proton}} n_b^{(0)}$. Hubble constant estimates were higher than today ($\sim 100 \mathrm{km/sec/Mpc}$). The matter density was then $\rho_M = \frac{3H_0^2}{8\pi G_N} \sim 10^{-29}\,\mathrm{g/cm^3} \Rightarrow n_b^{(0)} \sim 10^{-5}\,\mathrm{cm^{-3}}$
- Therefore $T^{(0)} \sim 10^9\,\mathrm{K}\left(\frac{n_b^{(0)}}{n_b^{(i)}}\right)^{1/3} \sim 20\,\mathrm{K}$
- In reality the number density of baryons today is $n_b^{(0)} \sim 10^{-7}\,\mathrm{cm}^{-3}$ which would give $T_{cmb} \sim 5$ K based on the above estimates
- $\Rightarrow$the Univese today **should be filled with radiation** whose spectrum peaks at $\lambda = \frac{2.9\,\mathrm{mm \cdot K}}{T}$

Formation of structures

- The Universe was hot (**radiation-dominated** at epoch of nucleosynthesis. But the density of radiation dropped faster than the density of matter:

$$\frac{\rho_{rad}}{\rho_b} \sim \frac{1}{a}$$

$\Rightarrow$**matter-radiation equality** (at $T \sim 10^3$ K!)

- The growth of Jeans instabilities did not start until that matter-dominated epoch (see below)

- Gamow estimates the size of the instability as

Paper by Gamow (1948

$$k_B T_{\text{eq}} \sim \frac{G_N \rho_{\text{matter}} R^3}{R}$$

Putting in the $T_{eq} \sim 10^3$ K one gets $R \sim 1$ kpc similar to a typical galaxy size(!)

Hot Big Bang theory was born

Predictions of Hot Big Bang model

- CMB
- Baryon-to-photon ratio from BBN and CMB (independently)
- Primordial abundance of light elements. Most notably, ^{4}He

Challenges to Hot Big Bang

In **1950s** this was not so obvious!

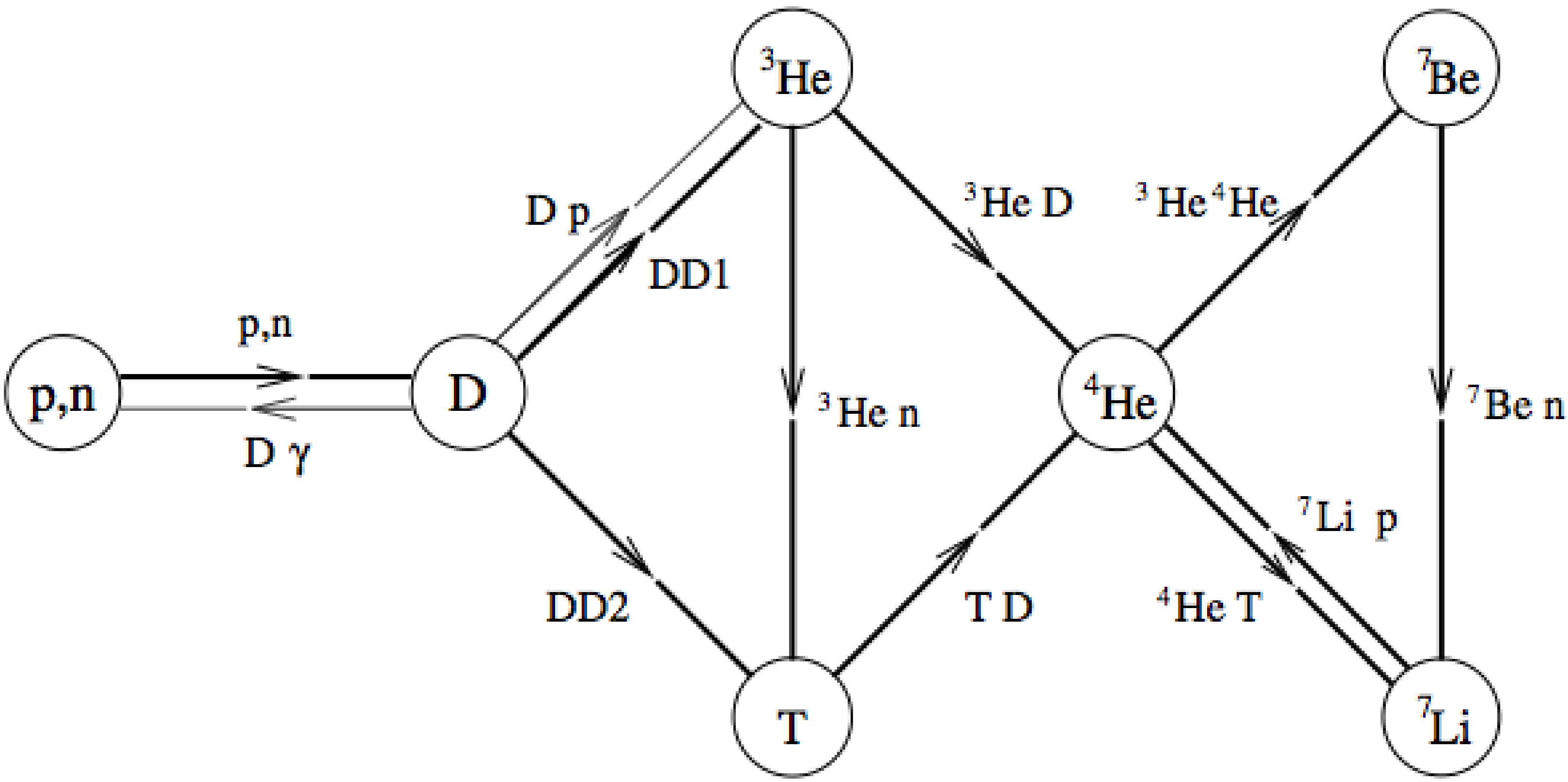
- There was no relict radiation from recombination
- You should get about 30% of Helium (which was considered to be wrong, as its abundance was measured $\sim 10\%$)
- In low density hot matter you cannot produce heavy nuclei ($A = 5$ and $A = 8$) in this way. With Hubble constant at that time $H_0 \sim 500\mathrm{km/sec/Mpc}$ the age of the Universe $\approx$ the age of the **Earth** $\Rightarrow$heavy elements could not be produced in stars, should be in the Universe "from the very beginning".

Problems 5c for $H_0 = 500$ km/sec/Mpc

It was concluded by many that "Hot Big Bang" is ruled out

see e.g. Zel'dovich UFN 1963

- To produce chemical elements one needs to pass through "deuterium bottleneck" $p + n \to D + \gamma$



Deuterium bottleneck

- Deuterium is the first element to produced. Reaction $p+n \to D+\gamma$
- We saw that for each baryon there were $\sim 10^{10}$ photons.
- Binding energy of deuterium is $E_D = 2.2$ MeV (or $T_D = 2.5\times 10^{10}$ K).
- At $T = E_D$ 85% of all photons have $E > T_D$ ⇒any deuterim nucleus will be quickly photo-disassociated via $D+\gamma \to p+n$
- Production of deuterium becomes efficient when temperature drops so that the number of photons with $E > E_D$ will be $\sim 10^{-10}$

$$\frac{n_\gamma(E > E_D)}{n_{\gamma_{tot}}} \sim \eta_B \Longrightarrow \eta_B \left(\frac{2.5 T_{BBN}}{m_p}\right)^{\frac{3}{2}} e^{\frac{E_D}{T_{BBN}}} \sim 1 \qquad (2)$$

$$T_{BBN} \approx 70\,\mathrm{keV} \quad \text{and} \quad t_{BBN} = \frac{M^*_{Pl}}{2T^2_{BBN}} \approx 120s$$

Neutron/proton ratio

- How many neutrons and protons are there (so far we did not distinguish between them)

- At high temperatures chemical equilibrium between protons and neutrons is maintained by weak interactions $n + \nu \leftrightarrows p + e^-$, $n + e^+ \leftrightarrows p + \bar{\nu}$, $n \leftrightarrows p + e^- + \bar{\nu}_e$

- Description of these processes is given by Fermi 4-fermion theory:

$$\mathcal{L}_{\text{Fermi}} = -\frac{G_F}{\sqrt{2}}[\bar{p}(x)\gamma_\mu(1-\gamma_5)n(x)][\bar{e}(x)\gamma^\mu(1-\gamma_5)\nu(x)] \qquad (3)$$

  Fermi coupling constant $G_F \approx 10^{-5}\,\mathrm{GeV}^{-2}$

- **Problem:** demonstrate the dimensionality of the Fermi coupling constant

The content of MeV plasma

- If temperatures was at least few MeV, we expect plasma to contain **electron-positron pairs** in equilibrium amounts ($\gamma + \gamma \leftrightarrows e^+ + e^-$ for $T \gtrsim m_e$)
- We also know that the plasma contained some number of protons and neutrons (their origin will be discussed later)
- Weak reactions were in equilibrium until $T \sim 1$ MeV

Many weak reactions that produce neutrinos (ν_e) are responsible for keeping p and n in thermal equilibrium

$$p + e^- \leftrightarrows n + \nu_e \quad n + e^+ \leftrightarrows p + \bar{\nu}_e$$
$$n \leftrightarrows p + e^- + \bar{\nu}_e$$

- Cross-section (in units length^2) in $2 \to 2$ reactions is defined as

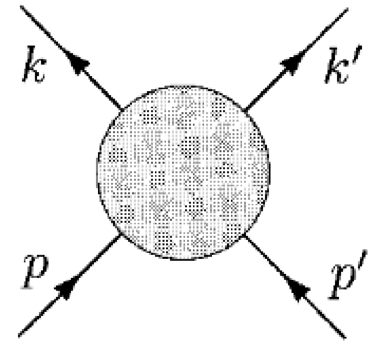
$$\sigma \sim \int \frac{d^3k}{(2E_p)(2\pi)^3}\frac{d^3k'}{(2E_{k'})(2\pi)^3}|\mathcal{M}|^2\delta^4(\sum_{in} p - \sum_{out} k)$$

  where $|\mathcal{M}|^2$ is a **matrix element** – probability of scattering – for a particular choice of incoming and outgoing momenta p_{in} and k_{out}

- cross-section can depend only on Lorentz invariant quantities
  - masses of particles
  - coupling constants
  - 3 Lorentz-invariant combinations of incoming and outgoing momenta, **Mandelstam variables**:

Cross-section, reaction rates

$$s = (p+p')^2 = (k+k')^2 \quad = 4E_{cm}^2$$
$$t = (k-p)^2 = (k'-p')^2$$
$$u = (k-p')^2 = (k'-p)^2$$



- If all incoming particles are relativistic, $E \gg m$, we expect that total cross-section is a function of center-of-mass energy only

- Example: QED

$$\sigma \sim \frac{\alpha^2}{E_{cm}^2}$$

the real answer e.g. for $e^+ + e^- \to \gamma + \gamma$ is given by $\sigma = \frac{\pi\alpha^2}{2E_{cm}^2}$ up to some $\log(E/m_e)$ corrections

- Example: Fermi theory. Coupling constant G_F has dimension

Cross-section, reaction rates

$[G_F] =$ GeV^{-2}, cross-section $[\sigma] =$ GeV^{-2}.

$$\sigma \sim G_F^2 E_{cm}^2$$

- To check whether these reactions are in equilibrium, compared scattering rate due to Fermi interaction with the Hubble expansion rate:

  $$\Gamma \sim \sigma n_\nu = G_F^2 E_{cm}^2 T^3 \sim G_F^2 T^5$$

  (as $E_{cm} \sim T$ for particles in thermal equilibrium)

- For reactions with neutrons and protons one should also take into account that they are not relativistic and their number density is given by Boltzmann distribution.

- These reactions go out of equilibrium at $T_\nu \approx 1$ MeV

- The difference of concentrations of n and p at that time is

$$\frac{n_n}{n_p} = \exp\left(-\frac{m_n - m_p}{T_\nu}\right) \approx \frac{1}{6}$$

$m_n - m_p = 1.2$ MeV

- Almost all neutrons will end up in ^{4}He. The **mass abundance of Helium** is

$$Y_p \equiv \frac{4n_{He}}{n_n + n_p} = \frac{4(n_n/2)}{n_n + n_p} = \frac{2(n_n/n_p)}{1 + n_n/n_p}$$

- If $\eta_B \sim 1$, Helium abundance would be $\frac{1/3}{1+1/6} \approx 0.28$
- However, as we saw due to $\eta_B \ll 1$ formation of deuterium (preceeding formation of Helium) does not happen until $T \sim 70$ keV
- Therefore there is a time-delay between freeze-out of weak reaction and time of Helium formation. The unstable neutrons (lifetime

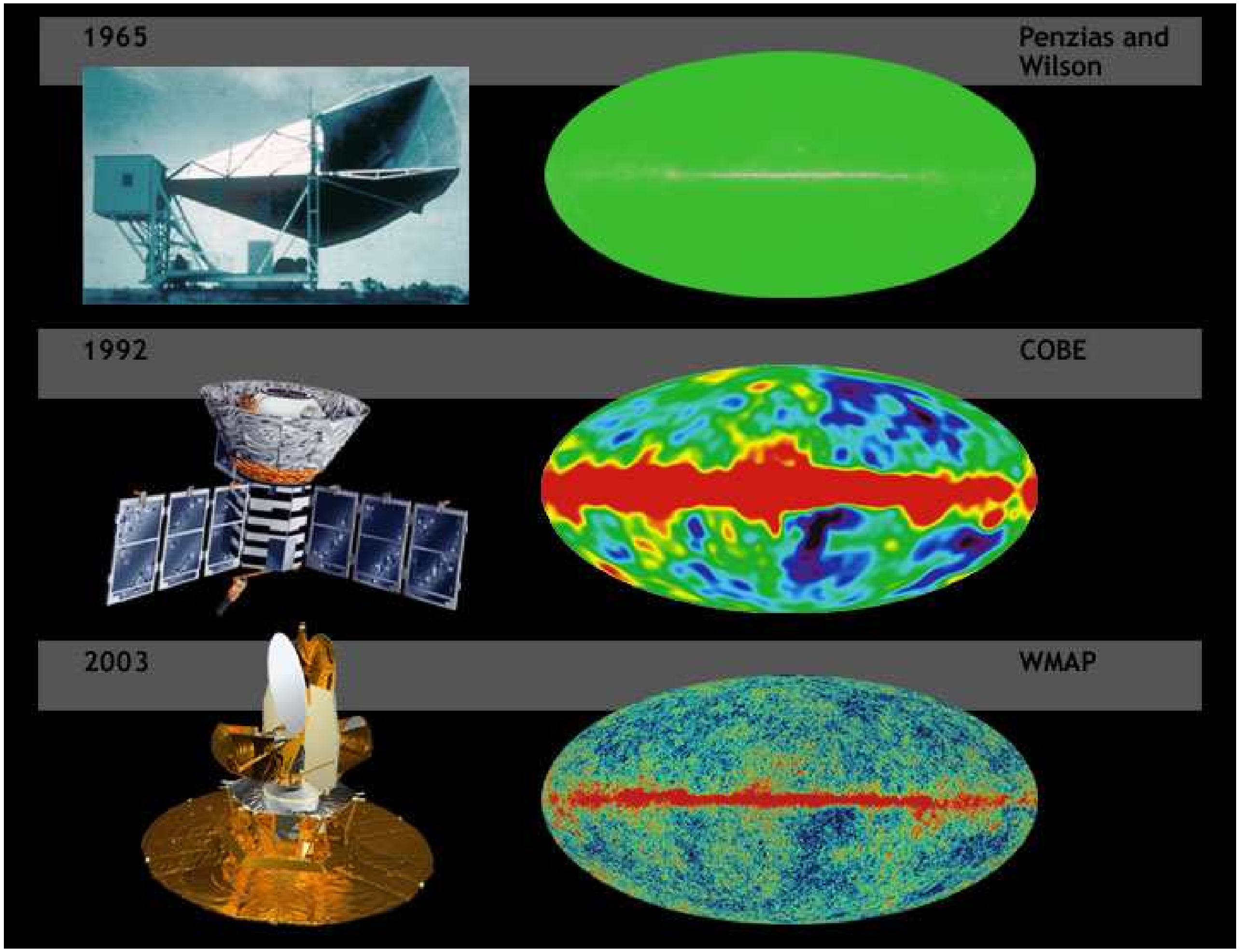
$\tau_n \sim 900$ sec decay and therefore by the time of Helium formation $n_n/n_p \approx 1/7$, which gives $Y_p \approx 25\%$

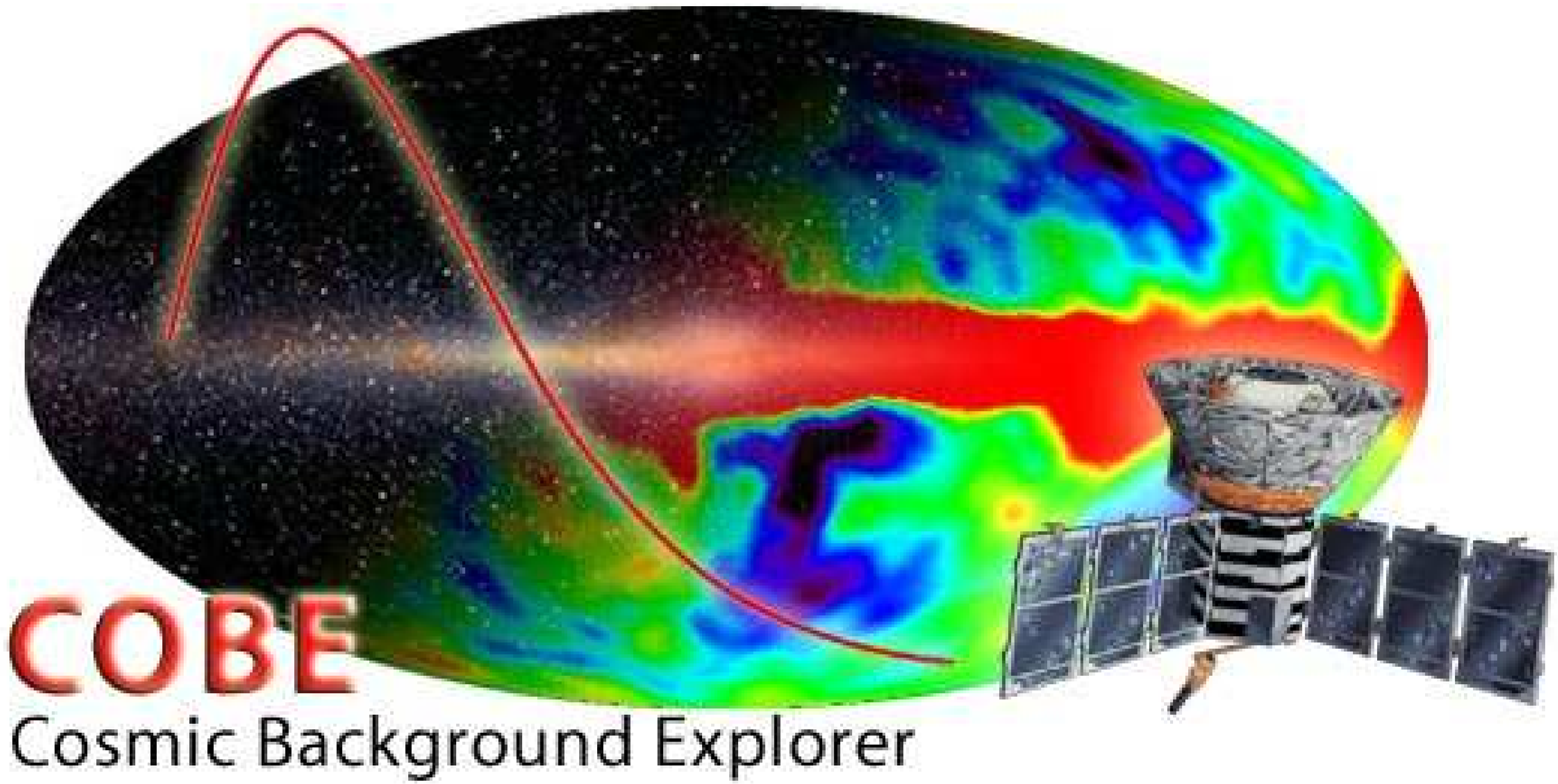
- $\Rightarrow$ ^{4}He is the second most abundant element in the Universe (after hydrogen)
- The Helium abundance is known with a precision of a few% (e.g. $Y_p = 0.2565 \pm 0.0010(\text{stat.}) \pm 0.0050(\text{syst,}))$ and is indeed very close to 25%

Izotov & Thuan (2010)

Cosmic Microwave background

Accidentally discovered by Arno Penzias and Robert Wilson: 1965

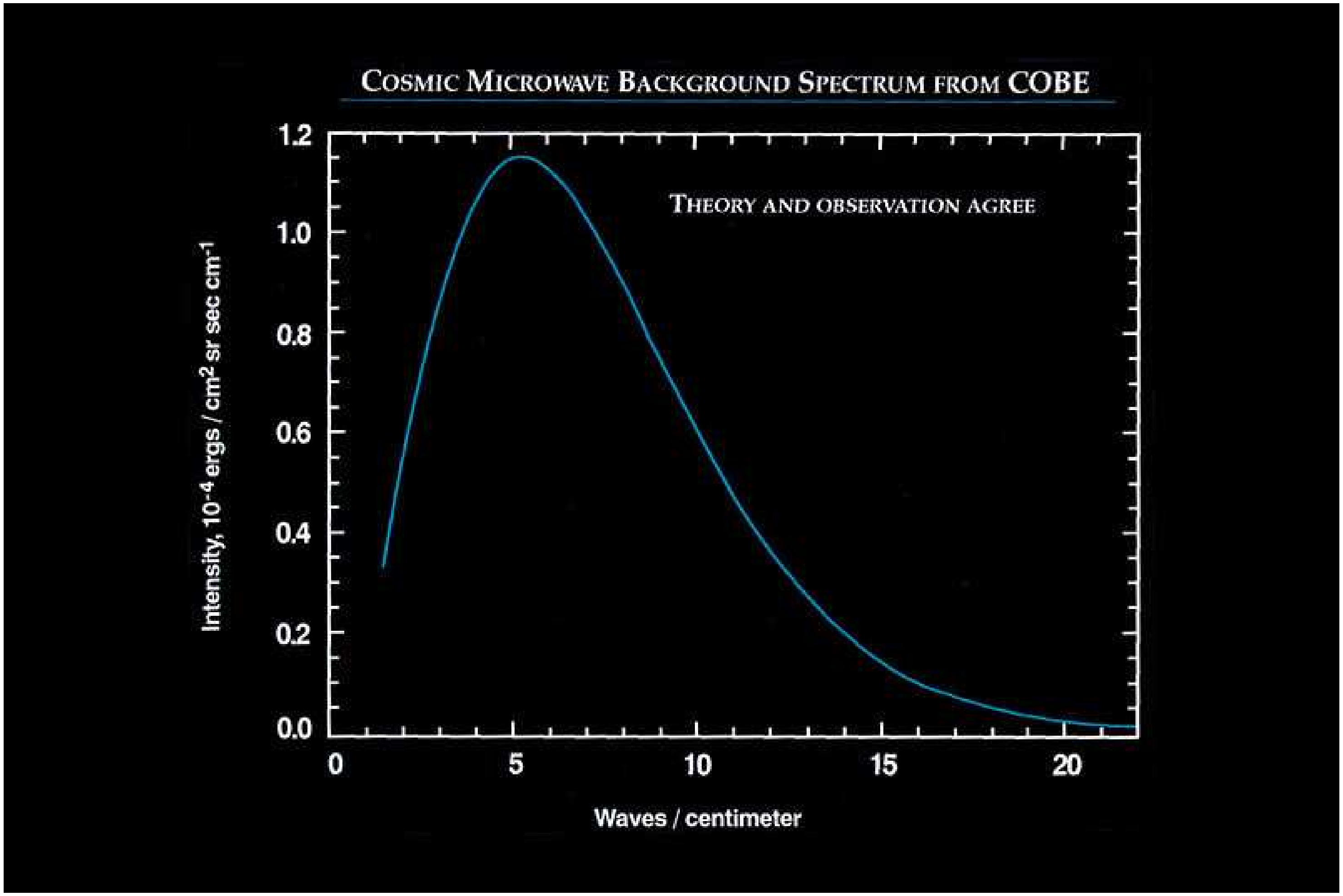




Data from **COBE** (1989 – 1996) showed a perfect fit between the black body curve and that observed in the microwave background.

CMB spectrum

Cosmic microwave background radiation is almost perfect **blackbody**



CMB temperature $T = 2.725\ K$

Properties of CMB

- Temperature of CMB $T = 2.725\ K$
- CMB contribution to the total energy density of the Universe:
$$\Omega_\gamma \simeq 4.5 \times 10^{-5}$$
- Spectrum peaks in the microwave range at a frequency of 160.2 GHz, corresponding to a wavelength of 1.9 mm.
- **410** photons per cubic centimeter
- *Almost* perfect blackbody spectrum ($\delta T/T < 10^{-4}$)
- COBE has detected anisotropies at the level $\delta T/T \sim 10^{-5}$

Go to CMB anisotropies section

- As temperature of the Universe drops, all protons will recombine with electrons to form neutral hydrogen: $e + p \to H + \gamma$. Binding energy $E_H = 13.6$ eV

- If $\eta_B \ll 1$ at $T \sim 13.6$ eV for each hydrogen atom there are many ionizing photons.

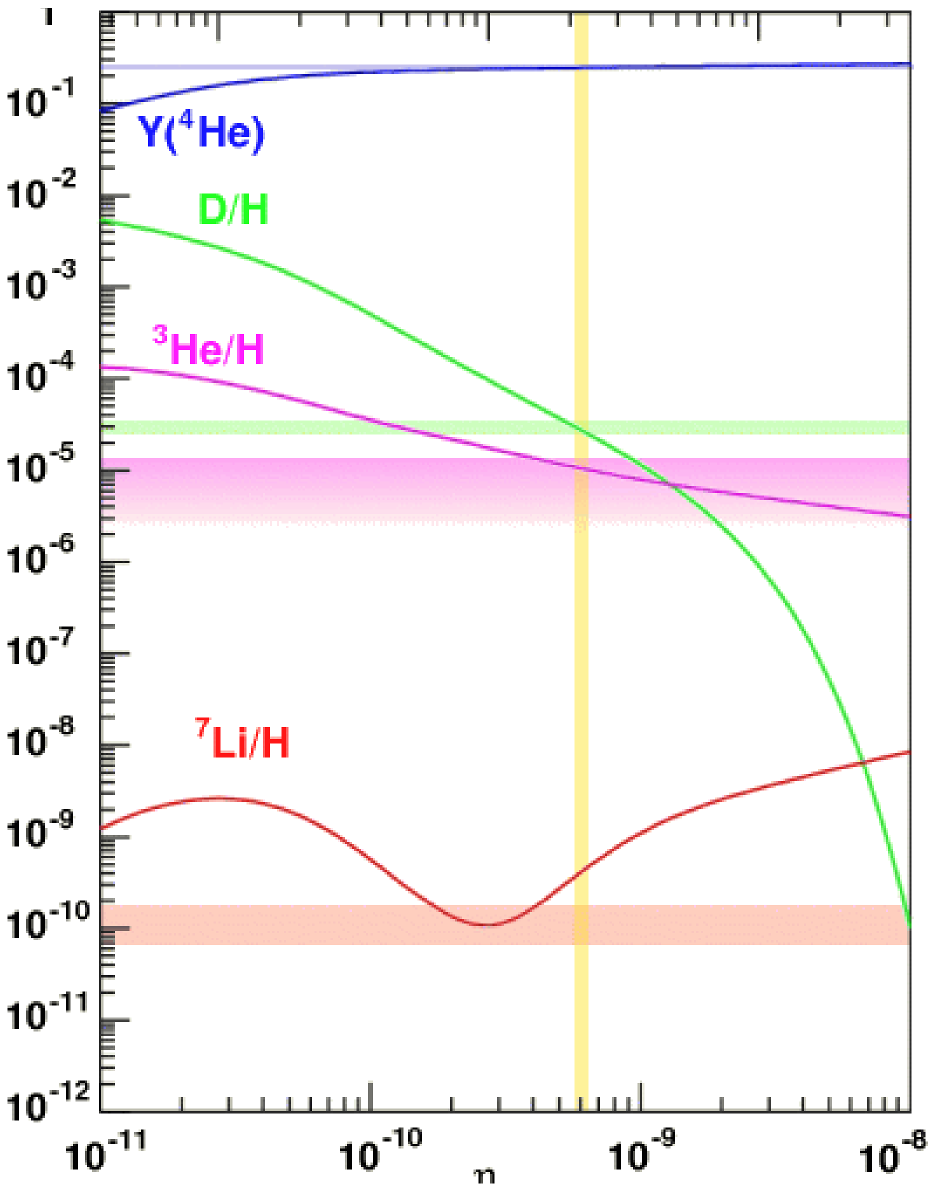
- As in the case of BBN and deuterium production, the temperature should drop significantly so that the number of energetic photons is small

- To find the number of “fast” photon, we describe high-energy tail of Bose-Einstein distribution as $f(k) \approx 1/(2\pi)^3 \exp[-k/T]$ and find the temperature when

$$\frac{n_\gamma(E > E_H)}{n_{\gamma,tot}} \approx \eta_B \tag{4}$$

- This gives the solution $T_{dec} \sim E_H/23 \approx 0.6$ eV
- Again, knowing T_{dec} and T_{cmb} today ($T_{cmb} = 2.725$ K), one can independently determine the baryon-to-photon ratio and **confirm** the BBN prediction

BBN predictions confirmed



- Curves – theoretical predictions of Big Bang nucleosynthesis
- Horizontal stripes – values that follow from observations.
- Golden stripe – measured value of η from CMB observations!

BBN and particle physics

Nowadays BBN has become a tool to determine properties (bounds) on light particles/decaying particles/evolution of fundamental constants

- The Helium abundance is known with a precision of a few% (e.g. $Y_p = 0.2565 \pm 0.0010(\text{stat.}) \pm 0.0050(\text{syst,}))$

Izotov & Thuan (2010)

- Neutron lifetime provides a “cosmic chronometer”, measuring the time between T_ν (**temperature of freeze-out of weak reactions**) and T_d (**temperature of deuteron production**):

$$\left.\frac{n_n}{n_p}\right|_{T_d} = \left.\frac{n_n}{n_p}\right|_{T_\nu} e^{-t/\tau_n}$$

- This time depends on the temperature T_d **and** number of relativistic species at that time

Effective number of relativistic d.o.f.

- Total energy density at radiation dominated epoch (i.e. the Hubble expansion rate/lifetime of the Universe) depends on the **effective number of relativistic degrees of freedom**:

$$\frac{3}{8\pi G_N}H^2 = \rho_{\text{rad}} = \frac{\pi^2}{30}g_* T^4 \quad \text{or}$$

where the number of relativistic degrees of freedom is given by

$$g_* = \sum_{\text{boson species}} g_i + \frac{7}{8}\sum_{\text{fermion species}} g_i$$

where relativistic species (having $\langle p\rangle \gtrsim m$) count

Problems 6b, 6d

BBN and particle physics

The primordial Helium abundance may change if

- There are more than 3 neutrino species (Roughly: one extra neutrino or a particle with similar energy density is allowed at about 2σ level)
- There was any other particle with the mass $\ll$ MeV and lifetime of the order of seconds or more that was contributing to g_* at BBN epoch (1 second – lifetime of the Universe at $T \sim 1$ MeV)
- There were heavy particles with lifetime in the range 0.01 – few seconds (that were decaying around BBN epoch)
- Newton's constant (entering Friedmann equation) changed between BBN epoch and later times (e.g. CMB or today)

NEUTRINO IN THE EARLY UNIVERSE

Neutrino properties

- there are 3 neutrinos (for each generation): ν_e, ν_μ, ν_τ
- neutrinos are stable
- neutrinos are electrically neutral
- neutrinos have tiny masses (much smaller than mass of the electron)
- neutrinos participate in weak interactions

How neutrinos are produced in the early Universe?

Neutrino reaction rates?

- Recall: weak interaction strength is **Fermi coupling constant** $G_F \approx 10^{-5}$ GeV^{-2}

- In the processes like $e^+ + e^- \to \nu_\alpha + \bar{\nu}_\alpha$ the interaction rate

$$\Gamma_{ee\to\nu\bar{\nu}} = n_e(T) \times \sigma_{\text{Weak}}$$

where

$$\sigma_{\text{Weak}} \propto G_F^2 \times E_e^2$$

What is the typical energy of electrons in this reaction?

- If in the expanding Universe particles that are in thermal equilibrium have either Fermi-Dirac or Bose-Einstein distributions

- At temperatures $T \gg m$ electron distribution function is

$$f_e(p) = 4\int \frac{d^3p}{(2\pi)^3}\frac{1}{e^{p/T}+1}$$

- Number density of the electrons

$$n_e(T) = 4\int \frac{d^3p}{(2\pi)^3}\frac{1}{e^{p/T}+1} \propto T^3$$

- Average energy of the electron $E_e = c \times \langle p \rangle$ i.e

$$E_e = \frac{4}{n_e(T)}\int \frac{d^3p}{(2\pi)^3}\frac{p}{e^{p/T}+1} \sim T$$

- As a result $E_e \sim T$
- Reaction rate $\Gamma_{ee\to\nu\bar{\nu}} \sim G_F^2 T^5$
- Compare the characteristic interaction time $\Gamma^{-1}_{ee\to\nu\bar{\nu}}$ with the age of the Universe $t_{\text{Univ}} = 1/H(T)$. To establish equilibrium we need $\Gamma^{-1}_{ee\to\nu\bar{\nu}} \ll t_{\text{Univ}}$ or $\Gamma_{ee\to\nu\bar{\nu}} \gg H(T)$

At what temperatures neutrinos are in equilibrium?

- One can see that temperature when

$$\Gamma \sim G_F^2 T^5 = T^2 \sqrt{\frac{8\pi G_N}{3} g_*(T)}$$

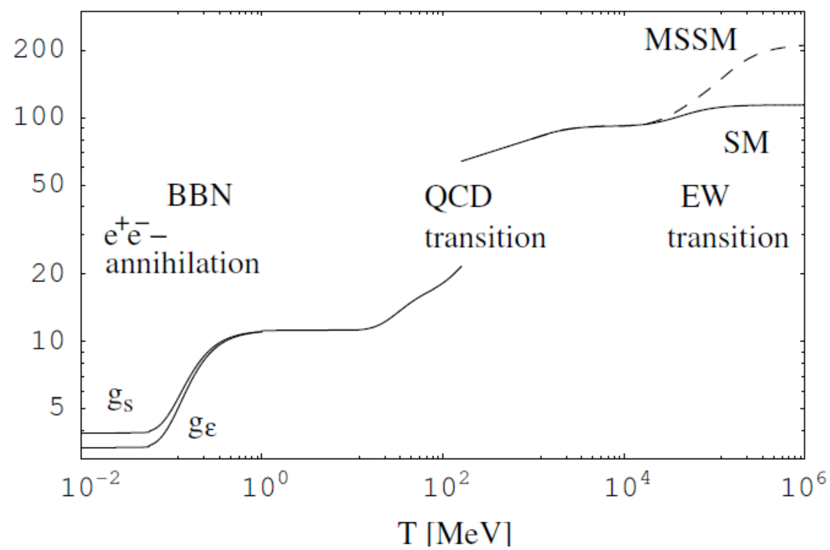
is roughly $T_{\text{dec}} \sim 1$ MeV

The Friedmanns equation for RD epoch can be written as:

$$H^2(T) = \frac{8\pi G_N}{3} \underbrace{g_*(T)\frac{\pi^2}{30}T^4}_{\rho_{\rm rad}}$$

where g_* – effective number of relativistic degrees of freedom.

As a result, $2 \lesssim g_* \lesssim 110$ for Standard Model:



Neutrino in the early Universe: summary

We saw that

- Neutrinos are produced in the early Universe and are in thermal equilibrium in plasma at $T \gtrsim T_{\text{dec}} \sim 1$ MeV
- As all equilibrium ultra-relativistic particles their average energy is $\langle E_\nu \rangle \sim T$, their number density is $\sim T^3$
- Their interaction rate with other particles $\Gamma_\nu \sim G_F^2 T^5$

What happens below T_{dec}?

Freeze-out

- If $\Gamma \lesssim H$ particle go out of thermal equilibrium – **freeze-out**.

- After the freeze-out, the comoving number density is conserved (particles are no longer produced or destroyed):

$$n_{\rm co}(T > T_{\rm dec}) = n_{\rm co}(T_{\rm dec}) \propto T_{\rm dec}^3$$

- The average momentum of decoupled particles changes with time (redshifts). Average momentum at the time of decoupling was ~ 1 MeV. Average momentum today is $\sim 10^{-3}$ eV

- As a result **today** in the Universe there are lots (about $100\ \mathrm{cm}^{-3}$) neutrinos (**exercise:** reproduce this number)

- Their **energy density** today:

$$\rho_\nu = \sum m_\nu \times n \quad \text{or numerically} \quad \Omega_\nu h^2 \approx \frac{\sum m_\nu}{94\ \text{eV}}$$